

$D = 4, \mathcal{N} = 1$ orientifolds with vector structure

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Abstract

We construct compact type IIB orientifolds with discrete groups \mathbf{Z}_4 , \mathbf{Z}_6 , \mathbf{Z}'_6 , \mathbf{Z}_8 , \mathbf{Z}_{12} and \mathbf{Z}'_{12} . These models are $\mathcal{N} = 1$ supersymmetric in $D = 4$ and have vector structure. The possibility of having vector structure in \mathbf{Z}_N orientifolds with even N arises due to an alternative Ω -projection in the twisted sectors. Some of the models without vector structure are known to be inconsistent because of uncancelled tadpoles. We show that vector structure leads to a sign flip in the twisted Klein bottle contribution. As a consequence, all the tadpoles can be cancelled by introducing $D9$ -branes and $D5$ -branes. To preserve supersymmetry, one has to choose an oscillator vacuum such that $\Omega^2|_{95} = +1$.

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1 Introduction

Four-dimensional $\mathcal{N} = 1$ supersymmetric compact type IIB orientifold models have been extensively studied in the past [1, 2, 3, 4, 5, 6, 7]. The usual construction is based on a product of the orbifold group Γ and the world sheet parity Ω such that the whole orientifold group is of the form: $\Gamma + \Omega\Gamma$. The action of the orientifold group on the Chan-Paton matrices is specified by choosing a projective real representation of the orbifold group Γ [8, 9]. It is known that there is a freedom in the choice of this representation, whenever Γ contains elements of even order (i.e. the smallest positive integer N , such that $g^N = e$, is even for some $g \in \Gamma$, where e is the neutral element of Γ). This freedom is related to the notion of vector structure in orientifold models, defined by the authors of [10]. It can also be understood from the theory of projective representations of finite groups (see e.g. [9]). For all Γ with only elements of odd order, one can always represent an element g satisfying $g^N = e$ by a matrix γ_g satisfying $\gamma_g^N = \mathbb{I}$. But if Γ contains elements of even order, there are two inequivalent choices for the representation matrix γ_g of an element g of even order N : $\gamma_g^N = \mu\mathbb{I}$, with $\mu = \pm 1$. Orientifold models with $\mu = +1$ ($\mu = -1$) have been called to have (no) vector structure in [10].

The six-dimensional \mathbb{Z}_2 orientifold of Gimon and Polchinski [11] has no vector structure.¹ However, motivated by F-theory orbifolds, it was found by Dabholkar and Park [12] and by Blum and Zaffaroni [13] that a $D = 6$ \mathbb{Z}_2 orientifold with vector structure can be obtained. This model (called DPBZ in the following) is realized by taking an orientifold group $\{1, R, \Omega(-1)^{F_R}R_1, \Omega(-1)^{F_L}R_2\}$, where R_i is the inversion of the i -th internal torus, $i = 1, 2$, and $R = R_1R_2$. In order to cancel the untwisted tadpoles, two types of $D7$ -branes are needed: $D7$ -branes wrapping the first torus and $D7'$ -branes wrapping the second torus. The closed string spectrum consists of 17 tensor multiplets and 4 hypermultiplets. After applying T-duality in the second torus one expects to get the Gimon-Polchinski (GP) model with an orientifold group $\{1, R, \Omega, \Omega R\}$. But the closed string spectrum of this latter model is quite different: it has only one tensor multiplet and 20 hypermultiplets. It turns out that the T-dual of the DPBZ model has a slightly different Ω projection as compared to the GP model: $\Omega_{\text{DPBZ}} = \Omega_{\text{GP}} T$, where the operator T flips the sign of the twisted fields at all fixed points. This change of sign in the twisted closed string modes is responsible for the enhancement of the number of tensor multiplets [12, 14].

Following the argument of [14], one can see that this change should be accompanied by a change in the consistency conditions on the action of the orientifold group elements on the

¹Note that in [11] the authors choose a basis such that $\gamma_R^2 = \mathbb{I}$, but γ_R is not real. A real representation is obtained by replacing $\gamma_R \rightarrow i\gamma_R$. This latter basis is used in [10] to show that this orientifold has no vector structure.

Chan-Paton matrices. That consistency condition is of the form:

$$\gamma_R = -\epsilon \gamma_\Omega \gamma_R^\top \gamma_\Omega^{-1}, \quad \text{with } \epsilon = \begin{cases} +1 & \text{for the GP } \Omega\text{-projection} \\ -1 & \text{for the DPBZ } \Omega\text{-projection} \end{cases} \quad (1.1)$$

It follows that in the GP model, γ_R has to be antisymmetric (and real) which implies $\gamma_R^2 = -\mathbb{I}$. However, in the DPBZ model, it is possible to take a representation with vector structure for the \mathbb{Z}_2 group. In order to get a supersymmetric model, one should correlate the closed string sign and the action of Ω^2 on the $77'$ sectors. All these changes produce a model with 32 $D7$ -branes and 32 $D7'$ -branes, with gauge group $SO(8)^4 \times SO(8)^4$ and without charged hypermultiplets [12, 13].

The aim of this article is to generalize the orientifold construction of [12, 13] to $D = 4$ and $\Gamma = \mathbb{Z}_N$. We analyse the models with even N , where the distinction between the two cases $\mu = \pm 1$, with and without vector structure, is relevant. All the features of the DPBZ model discussed above also appear in the four-dimensional models. In particular there is a curious interplay between different factors that may appear in type IIB orientifold constructions: vector structure μ , action of Ω on mixed sectors and the discrete torsion.²

The relation between the action of Ω^2 on the mixed sectors, the vector structure μ_p , $p = 9, 5$, and the sign ϵ defined in (1.1) can be summarized by

$$\Omega^2|_{95} = \mu_9 = \mu_5 = -\epsilon. \quad (1.2)$$

The first equation is needed for the model to be supersymmetric. If one is interested in studying models with vector structure and insists in keeping the GP action of Ω^2 on the mixed sectors ($\Omega^2|_{95} = -1$), some anti- $D5$ -branes will be needed to cancel the untwisted tadpoles [17, 16, 19, 18]. Another possibility to keep $\Omega^2|_{95} = -1$, is to consider orientifolds with two different kinds of fixed points, with and without vector structure [14]. This reduces the number of $D5$ -branes.

When trying to construct four-dimensional type IIB orientifold models that correspond to discrete groups Γ which would lead to $D = 4$, $\mathcal{N} = 1$ heterotic orbifolds, the authors of [5, 6] found that some of these orientifolds are inconsistent. Consider the simplest model that presents this kind of inconsistency: the \mathbb{Z}_4 orientifold [6]. The problem appears in the tadpole cancellation condition of the order-two twisted sector. The fixed set in this sector consists of 16 tori. Four of them are located at \mathbb{Z}_4 fixed points in the first two planes. The \mathbb{Z}_4 group acts as a \mathbb{Z}_2 inversion on each of these four tori and permutes the remaining 12 tori. The \mathbb{Z}_4 -invariant set is given by four T^2/\mathbb{Z}_2 's and 6 pairs of tori. There is a contribution

²Discrete torsion is not possible for the models discussed in this article, but only for $\mathbf{Z}_N \times \mathbf{Z}_M$ orientifolds. These will be analysed in [15]. For a discussion of the various phase factors see [9, 15].

from the Klein bottle to the order-two twisted tadpoles at each of the four \mathbb{Z}_4 fixed points in the first two planes proportional to

$$\frac{16}{V_3}(16 + \epsilon 16) \quad (1.3)$$

where V_3 is the volume of the third torus and ϵ is defined in (1.1). This contribution can be interpreted as the sum of the twisted charges of the $O9$ -plane and the 16 $O5$ -planes that wrap the third torus and sit at 16 \mathbb{Z}_2 fixed points in the first two tori. The case without vector structure ($\mu = -\epsilon = -1$) has a non-vanishing contribution from the orientifold planes that can not be cancelled by adding any set of D -branes because in this sector the twisted charge of the D -branes vanishes. Only in the non-compact case, when $V_3 \rightarrow \infty$, and in the case with vector structure ($\mu = -\epsilon = 1$) does this tadpole vanish. The impossibility to cancel this tadpole contribution using the standard GP Ω -projection can also be seen in the effective field theory which suffers from non-Abelian gauge anomalies. The same inconsistency was found in the \mathbb{Z}_8 , \mathbb{Z}'_8 and \mathbb{Z}'_{12} orientifolds without vector structure [6].

Similarly, Zwart found [5] that the $\mathbb{Z}_N \times \mathbb{Z}_M$ orientifolds, with N or M a multiple of four, are not consistent. These models have discrete torsion in the sense of [20, 9] and use the standard GP Ω -projection (i.e. have no vector structure in $D9$ and three sets of $D5$ -brane sectors). This inconsistency is not related to uncanceled tadpoles but rather to the algebra of the representations.

However, \mathbb{Z}_N , N even, orientifolds with vector structure can be consistently constructed for the discrete groups \mathbb{Z}_4 , \mathbb{Z}_6 , \mathbb{Z}'_6 , \mathbb{Z}_8 , \mathbb{Z}_{12} and \mathbb{Z}'_{12} .³ Probably the \mathbb{Z}'_8 orientifold also exists, although we were not able to find a consistent solution. In section 2 we discuss the construction of these models: we analyse the closed and open string spectrum and the tadpole cancellation conditions. The physical interpretation of the various signs that appear in this computation leads us to speculate about the existence of D -branes with negative NSNS charge. In section 3 we give an explicit solution of the tadpole conditions for each of the above orientifolds. We concentrate on the brane configurations leading to the gauge group with maximal rank. This corresponds to locating a maximum number of $D5$ -branes at the \mathbb{Z}_N fixed points in such a way that all the tadpoles are cancelled. In some cases not all the $D5$ -branes can be put at the origin but some of them will be needed at other fixed points to cancel the Klein bottle contribution. For the \mathbb{Z}_8 and \mathbb{Z}_{12} orientifold, it is necessary to put some $D5$ -branes at points which are not fixed under \mathbb{Z}_N . We verify the correspondence between the tadpole cancellation conditions and the non-Abelian gauge anomalies that was expected from the general analysis of the models without vector structure[21, 22]. The

³Furthermore, it is also possible to construct $\mathbb{Z}_N \times \mathbb{Z}_M$ orientifolds, with N or M a multiple of four, as will be shown in [15].

Abelian gauge anomalies are expected to be cancelled by the four-dimensional generalization of the Green-Schwarz mechanism [24, 25]. In the two appendices we explain how to obtain the open string spectrum using orientifold quivers and sketch a derivation of the tadpole cancellation conditions including all possible phase factors.

2 Construction of the models

We consider compact orientifolds of the form $T^6/(\Gamma \times \{\mathbb{1}, \Omega\})$, with $\Gamma = \mathbb{Z}_N$, N even. The six-torus is defined as $T^6 = \mathbb{C}^3/\Lambda$, with Λ a factorizable lattice, i.e. it is the direct sum of three two-dimensional lattices.⁴ The world-sheet symmetry Ω is of the form

$$\Omega = \Omega_0 J T, \quad (2.1)$$

where Ω_0 is the world-sheet parity, the operator J exchanges the k -th and the $(N - k)$ -th twisted sector [14] and T acts as $-\mathbf{1}$ on all twisted states. The necessity of the additional operator T is related to the fact that the models considered in this article have vector structure. This operator was discussed by the authors of [12, 14] when analysing a new \mathbb{Z}_2 orientifold in $D = 6$.

The action of Γ on the coordinates (z_1, z_2, z_3) of \mathbb{C}^3 can be characterized by the shift vector $v = (v_1, v_2, v_3)$:

$$g_1 : z_i \longrightarrow e^{2\pi i v_i} z_i, \quad (2.2)$$

where g_1 is the generator of Γ and $\sum_{i=1}^3 v_i = 0$ to ensure $\mathcal{N} = 1$ supersymmetry in $D = 4$. Not all possible shifts correspond to a symmetry of some lattice. Indeed, there is a finite number of \mathbb{Z}_N orbifolds (see e.g. [23]). For even N , there are only 7 models, table 1. The shift vectors are chosen such that the order-two twist $g_1^{N/2}$ fixes the third complex plane. This will require the introduction of $D5_3$ -branes when constructing the corresponding orientifolds.

The \mathbb{Z}_N orientifolds without vector structure have been discussed in [4, 6]. There it was found that the models with discrete groups \mathbb{Z}_4 , \mathbb{Z}_8 , \mathbb{Z}'_8 and \mathbb{Z}'_{12} are not consistent because they have uncanceled tadpoles. We will see that there are solutions to the tadpole equations for all of the models with vector structure. However, in the construction of the \mathbb{Z}'_8 orientifold, we encountered a difficulty and could not construct a consistent model.

⁴More precisely, we only need that for each $g \in \Gamma \setminus \{e\}$ that has fixed planes, the lattice Λ can be decomposed in a direct sum of sublattices $\Lambda = I \oplus J$, such that I is fixed under g and this decomposition is preserved under g [23]. The orientifolds corresponding to $\Gamma = \mathbb{Z}_8$, \mathbb{Z}'_8 , \mathbb{Z}_{12} , \mathbb{Z}'_{12} only satisfy this weaker condition.

Γ	v	Γ	v
\mathbb{Z}_4	$\frac{1}{4}(1, 1, -2)$		
\mathbb{Z}_6	$\frac{1}{6}(1, 1, -2)$	\mathbb{Z}'_6	$\frac{1}{6}(1, -3, 2)$
\mathbb{Z}_8	$\frac{1}{8}(1, 3, -4)$	\mathbb{Z}'_8	$\frac{1}{8}(1, -3, 2)$
\mathbb{Z}_{12}	$\frac{1}{12}(1, -5, 4)$	\mathbb{Z}'_{12}	$\frac{1}{12}(1, 5, -6)$

Table 1: Possible \mathbf{Z}_N orbifolds with even N and $\mathcal{N} = 1$ in $D = 4$.

In this section, we sketch the basic steps to construct \mathbb{Z}_N orientifolds with vector structure. We explain how to obtain the closed string spectrum, the open string spectrum and the tadpole cancellation conditions.

2.1 Closed string spectrum

The closed string spectrum can be obtained from the cohomology of the internal orbifold space, table 2. This is explained in [25, 9]. Let us summarize this method and analyse what changes if the Ω -projection (2.1) of Dabholkar, Park, Blum and Zaffaroni (DPBZ) [12, 13] is taken instead of the standard Ω -projection of Gimon and Polchinski (GP) [11].

We can split the sectors into three different types:

(i) The untwisted sector ($k = 0$), it is invariant under J and T . The bosonic fields in $D = 4$ are found contracting the Lorentz indices of the J -even 10D fields $g_{\mu\nu}$, ϕ , $C_{\mu\nu}^{(2)}$ with the harmonic forms corresponding to $h^{0,0}$, $h^{3,0}$, $h_{\text{untw}}^{1,1}$, $h_{\text{untw}}^{2,1}$.

(ii) The order-two sector ($k = N/2$), it is invariant under J but acquires an extra minus sign under the action of T . In this sector, one has to contract the J -odd 10D fields $B_{\mu\nu}$, $C_{\mu\nu\rho\sigma}^{(4)}$ with $h_{N/2}^{1,1}$, $h_{N/2}^{2,1}$. Only in this sector does the spectrum differ from the one obtained when using the standard GP Ω -projection.

(iii) The remaining sectors. To get the fields in $D = 4$, one forms linear combinations of the harmonic forms that belong to the k -th and $(N - k)$ -th twisted sector. The J -even forms are contracted with the J -odd 10D fields and the J -odd forms are contracted with the J -even 10D fields.

The spectrum fits into $\mathcal{N} = 1$ supermultiplets. In total, one finds:

(i) the gravity multiplet, a linear multiplet, $(h^{1,1} + h^{2,1})_{\text{untw}}$ chiral multiplets.

(ii) $h_{N/2}^{1,1}$ linear multiplets, $h_{N/2}^{2,1}$ vector multiplets.

Γ		$k = 0$	$k = 1$	$k = 2$	$k = 3$	$k = 4$	$k = 5$	$k = 6$	TOTAL
\mathbb{Z}_4	$h^{1,1}$	5	16	10	—	—	—	—	31
	$h^{2,1}$	1	0	6	—	—	—	—	7
\mathbb{Z}_6	$h^{1,1}$	5	3	15	6	—	—	—	29
	$h^{2,1}$	0	0	0	5	—	—	—	5
\mathbb{Z}'_6	$h^{1,1}$	3	12	6	8	—	—	—	35
	$h^{2,1}$	1	0	3	4	—	—	—	11
\mathbb{Z}_8	$h^{1,1}$	3	8	3	8	6	—	—	31
	$h^{2,1}$	1	0	1	0	4	—	—	7
\mathbb{Z}'_8	$h^{1,1}$	3	4	10	4	6	—	—	27
	$h^{2,1}$	0	0	0	0	3	—	—	3
\mathbb{Z}_{12}	$h^{1,1}$	3	3	3	2	9	3	4	29
	$h^{2,1}$	0	0	0	1	0	0	3	5
\mathbb{Z}'_{12}	$h^{1,1}$	3	4	1	8	3	4	4	31
	$h^{2,1}$	1	0	0	0	2	0	2	7

Table 2: Contribution from each sector to the orbifold Hodge numbers ($k = 0$ is the untwisted sector). The sectors corresponding to twists $k > N/2$ are not displayed. In the last column all the contributions are summed up (including untwisted and $k > N/2$ sectors).

(iii) for each $0 < k < N/2$: $h_k^{1,1}$ linear multiplets, $h_k^{2,1}$ vector multiplets; if the k -th sector has fixed planes, then there are additional $(h_k^{1,1} + h_k^{2,1})$ chiral multiplets.

2.2 Open string spectrum

There are 32 $D9$ -branes and 32 $D5_3$ -branes, the index 3 indicating that the 5-branes fill the four non-compact directions and the third complex plane. This is a consequence of the untwisted tadpole cancellation conditions to be discussed below.

The action of Γ on the Chan-Paton indices of the open strings is described by a (projective) representation $\gamma^{(p)}$ that associates a (32×32) -matrix $\gamma_{g,p}$ to each element g of Γ , where $p = 9, 5$ denotes the type of the D -brane the open string ends on.

$$\gamma^{(p)} : \Gamma \longrightarrow GL(32, \mathbb{C}) \quad (2.3)$$

$$g \longmapsto \gamma_{g,p}$$

Because of the orientifold projection, this representation must be real or pseudo-real. In general, $\gamma^{(p)}$ can be decomposed in irreducible blocks of real (R^r), pseudo-real (R^p) and complex (R^c) representations [8]:

$$\gamma^{(p)} = \left(\bigoplus_{l_1} n_{l_1}^r R_{l_1}^r \right) \oplus \left(\bigoplus_{l_2} n_{l_2}^p R_{l_2}^p \right) \oplus \left(\bigoplus_{l_3} n_{l_3}^c (R_{l_3}^c \oplus \bar{R}_{l_3}^c) \right). \quad (2.4)$$

In this expression, the notation $n_l R_l$ is short for $R_l \otimes \mathbb{I}_{n_l}$, i.e. n_l is the number of copies of the irreducible representation (irrep) R_l in $\gamma^{(p)}$ [8].

Let us first consider the 99 and 5₃5₃ sectors. The projection on invariant states of the Chan-Paton matrices $\lambda^{(0)}$ that correspond to gauge bosons in $D = 4$ imposes the constraints

$$\lambda^{(0)} = \gamma_{g_1,p} \lambda^{(0)} \gamma_{g_1,p}^{-1}, \quad \lambda^{(0)} = -\gamma_{\Omega,p} \lambda^{(0)\top} \gamma_{\Omega,p}^{-1}, \quad (2.5)$$

where g_1 is the generator of Γ . These are easily solved. One finds the gauge group [8]

$$G_{(p)} = \prod_{l_1} SO(n_{l_1}^r) \times \prod_{l_2} USp(n_{l_2}^p) \times \prod_{l_3} U(n_{l_3}^c) \quad (2.6)$$

from the Dp -branes if $\gamma_{\Omega,p}$ is symmetric and

$$G_{(p)} = \prod_{l_1} USp(n_{l_1}^r) \times \prod_{l_2} SO(n_{l_2}^p) \times \prod_{l_3} U(n_{l_3}^c) \quad (2.7)$$

if $\gamma_{\Omega,p}$ is antisymmetric.

In our case $\Gamma = \mathbb{Z}_N$. There are N irreps, all of them one-dimensional:

$$R_0(g_1) = 1, \quad R_1(g_1) = e^{2\pi i/N}, \quad \dots, \quad R_{N-1}(g_1) = e^{2\pi i(N-1)/N}. \quad (2.8)$$

These irreps have vector structure because $(R_l)^N = 1 \forall l$. For even N , R_0 and $R_{N/2}$ are real and the remaining irreps can be organized in pairs of conjugate representations: $R_l = \bar{R}_{N-l}$. Furthermore, we will see below that both $\gamma_{\Omega,9}$ and $\gamma_{\Omega,5}$ are symmetric. Thus the gauge group is

$$G_{(9)} = G_{(5)} = SO(n_0) \times SO(n_{N/2}) \times \prod_{l=1}^{N/2-1} U(n_l). \quad (2.9)$$

The matter fields corresponding to the i -th complex plane are obtained from the projections

$$\lambda^{(i)} = e^{2\pi i v_i} \gamma_{g_1,p} \lambda^{(i)} \gamma_{g_1,p}^{-1}, \quad \lambda^{(i)} = R_{\Omega}^{(i)} \gamma_{\Omega,p} \lambda^{(i)\top} \gamma_{\Omega,p}^{-1}, \quad (2.10)$$

$$\text{with } R_{\Omega}^{(i)} = \begin{cases} -1 & \text{if } p = 9 \text{ or } i = 3 \\ +1 & \text{if } p = 5 \text{ and } i \neq 3 \end{cases}.$$

These equations are solved in two steps. First, one draws the quiver diagram of the corresponding \mathbb{Z}_N orbifold [26]. This solves the first condition of (2.10). Second, the orientifold projection is performed on this quiver diagram, as explained in appendix A. The spectrum can be directly read off from the resulting orientifold quiver.

Let us now consider the 95_3 sector. The projection of the open strings $\lambda^{(95)}$, stretching from 9-branes to 5-branes, on Γ -invariant states reads (see e.g. [6]):

$$\lambda^{(95)} = e^{\pi i v_3} \gamma_{g_1, 9} \lambda^{(95)} \gamma_{g_1, 5}^{-1} \quad (2.11)$$

Here, we used $\sum_{i=1}^3 v_i = 0$ to transform $e^{2\pi i(-1/2)(v_1+v_2)} = e^{\pi i v_3}$. The additional minus sign in the exponent (cf. eq. (2.25) of [6]) is related to a different oscillator ground state in the 95_3 sector compared to the standard orientifold constructions. Dabholkar and Park showed that one has to choose the oscillator ground of the 95 sector of their $D = 6$ model such that the order-two twist $g_1^{N/2}$ acts as $-\mathbf{1}$ on this ground state. This leads to a sign flip in the projection of the 95 strings on invariant states and directly translates to the above formula for $D = 4$ orientifolds. Ω relates the 95_3 sector with the $5_3 9$ sector and does not impose extra conditions on $\lambda^{(95)}$. Again, the spectrum is easiest obtained using quiver theory, see appendix A.

2.3 Tadpoles

In this paragraph we give the tadpole cancellation conditions keeping track of all possible signs. This enables us to see how the tadpole equations of the models with DPBZ projection differ from those with GP projection. The derivation of these equations is sketched in appendix B.

It is convenient to label the elements of \mathbb{Z}_N by the integer $k = 0, \dots, N-1$, i.e. $k = 0$ denotes the neutral element and $k = 1$ the generator of \mathbb{Z}_N . We define $s_i = \sin(\pi k v_i)$, $c_i = \cos(\pi k v_i)$ and $\tilde{s}_i = \sin(2\pi k v_i)$. Let us comment on the various signs appearing in the tadpole contributions.

- In the cylinder amplitude, α weights the 95_3 sector relative to the 99 and $5_3 5_3$ sectors. In supersymmetric models, it is related to the action of Ω^2 on the oscillator part of the strings stretching from 9-branes to 5-branes by $\Omega^2|_{95_3} = -\alpha$. The authors of [11] argued that $\Omega^2|_{95_3} = -1$, but in [12, 13] it was shown that one can choose a different oscillator ground state in the 95_3 sector, such that $\Omega^2|_{95_3} = +1$.
- In the Klein bottle amplitude, ϵ is related to the choice between the standard and alternative Ω -projection. One has $\epsilon = +1$ for GP and $\epsilon = -1$ for DPBZ.⁵

⁵The same sign ϵ also appears in the non-supersymmetric models of [19].

- In the Möbius strip, μ_p and c_p are defined by $(\gamma_{1,p})^N = \mu_p \mathbb{I}$ and $\gamma_{\Omega,p}^\top = c_p \gamma_{\Omega,p}$. Models with $\mu_p = +1$ (-1) are said to have (no) vector structure.

A more precise definition of these signs is given in appendix B. For a physical interpretation, see section 2.4.

Factorization of the twisted tadpoles requires that

$$\epsilon = -\mu_9 = -\mu_5. \quad (2.12)$$

This relates the sign ϵ of the twisted states and the vector structure. One can easily check using the orientifold relations and the unitarity of the matrices that this condition is equivalent to the consistency condition (1.1) mentioned by Polchinski in [14].

The untwisted tadpoles can only be cancelled if

$$\alpha = \epsilon. \quad (2.13)$$

According to the interpretation of α and ϵ given in section 2.4, this is the statement that the D -brane charges must be opposite to the O -plane charges.

Furthermore, it is easy to see [11] that the action of Ω^2 on the oscillator part of strings stretching from 9-branes to 5-branes is related to c_9 and c_5 by

$$\Omega^2|_{95_3} = c_9 c_5 = \begin{cases} -1 & \text{for the standard oscillator ground state of GP} \\ +1 & \text{for the alternative oscillator ground state of DPBZ} \end{cases}. \quad (2.14)$$

The last equality follows from the arguments given in [11, 12].

A supersymmetric solution does only exist if

$$c_9 = 1, \quad \Omega^2|_{95_3} = -\alpha. \quad (2.15)$$

We are interested in the supersymmetric models with vector structure, i.e.

$$\Omega^2|_{95_3} = c_9 = c_5 = \mu_9 = \mu_5 = 1, \quad \epsilon = \alpha = -1. \quad (2.16)$$

The tadpole cancellation conditions for supersymmetric models are:

a) untwisted sector

$$\text{Tr } \gamma_{0,9} = 32, \quad \text{Tr } \gamma_{0,5} = 32 c_5 \mu_5 \quad (2.17)$$

b) twisted sectors without fixed tori, i.e. $kv_i \neq 0 \bmod \mathbb{Z}$:

- odd k :

$$\text{Tr } \gamma_{k,9} + 4 \alpha s_1 s_2 \text{Tr } \gamma_{k,5} = 0 \quad (2.18)$$

- even $k = 2k'$:

$$\text{Tr } \gamma_{2k',9} + 4 \alpha \tilde{s}_1 \tilde{s}_2 \text{Tr } \gamma_{2k',5} = 32 (c_1 c_2 c_3 + \epsilon s_1 s_2 c_3), \quad (2.19)$$

where s_i, c_i, \tilde{s}_i are evaluated with the argument k' .

c) twisted sectors with fixed tori, i.e. $kv_i = 0 \bmod \mathbb{Z}$:

- odd k :

- $i = 3$:

$$\text{Tr } \gamma_{k,9} + 4 \alpha s_1 s_2 \text{Tr } \gamma_{k,5} = 0 \quad (2.20)$$

- $i \neq 3$: never happens

- even $k = 2k'$, with $k'v_i = 0$:

- $i = 3$:

$$\text{Tr } \gamma_{2k',9} + 4 \alpha \tilde{s}_1 \tilde{s}_2 \text{Tr } \gamma_{2k',5} = 32 (c_1 c_2 + \epsilon s_1 s_2) \quad (2.21)$$

- $i \neq 3$:

$$\text{Tr } \gamma_{2k',9} = -32 c_3^2, \quad \text{Tr } \gamma_{2k',5} = -8 c_5 \mu_5 \quad (2.22)$$

- even $k = 2k'$, with $k'v_i = \pm \frac{1}{2}$:

- $i = 3$:

$$\text{Tr } \gamma_{2k',9} + 4 \alpha \tilde{s}_1 \tilde{s}_2 \text{Tr } \gamma_{2k',5} = 0, \quad \epsilon = -1 \quad (2.23)$$

- $i \neq 3$:

$$\text{Tr } \gamma_{2k',9} = \mp 32 \mu_9 c_3^2, \quad \text{Tr } \gamma_{2k',5} = \mp 8 c_5 \quad (2.24)$$

Note that the second condition in (2.23) can only be satisfied in models with vector structure, $\mu_9 = \mu_5 = 1$. This tadpole arises whenever $v_3 = \pm \frac{1}{2p}$ for some integer p , which is only possible for N a multiple of 4 because we want the third torus to be fixed under $k = N/2$. A short look at the shift vectors of table 1 shows that the orientifolds with discrete groups $\mathbb{Z}_4, \mathbb{Z}_8, \mathbb{Z}'_8$ and \mathbb{Z}'_{12} have this property. Therefore, they are only consistent with vector structure.

The $D5_3$ -branes may be distributed over different points in the first two internal tori. Of course the tadpole equations depend on the location of the 5-branes and not all configurations are consistent. Three different types of points have to be distinguished:

- \mathbb{Z}_N fixed points, like the origin.
- \mathbb{Z}_M fixed points (with M a divisor of N) which are not fixed under \mathbb{Z}_N .

- Points in the bulk.

The Klein bottle contribution to the tadpoles of the k -twisted sector consists of an untwisted part $\mathcal{K}_0(k)$ and a twisted part $\mathcal{K}_1(k)$. The former gives the term proportional to $c_1 c_2 c_3$ in (2.19), the latter the term proportional to $\epsilon s_1 s_2 c_3$. (Note that these terms are combined with the cylinder contribution to the $2k$ -twisted sector.) At each point of the internal space, the contribution $\mathcal{K}_0(k)$ is only present if this point is fixed under k and the contribution $\mathcal{K}_1(k)$ is only present if this point is fixed under $k + N/2$. Thus, the above tadpole cancellation conditions are strictly valid only at the \mathbb{Z}_N fixed points. At \mathbb{Z}_M fixed points which are not fixed under \mathbb{Z}_N , these conditions have to be modified accordingly.

One has to analyse the tadpoles at all the fixed points in each twisted sector. At each fixed point all the $D9$ -branes contribute but only those $D5_3$ -branes that are located at this point. The Klein bottle contribution to this fixed point is determined as explained in the preceeding paragraph. We will see how this works in the examples below. In the some of the models it is not possible to put all the $D5_3$ -branes at the origin.

The twisted tadpole conditions from sectors that do not fix the third torus are in one-to-one correspondence to the conditions arising from the requirement of absence of non-Abelian gauge anomalies. This correspondence has been studied for orientifolds with GP projection in [21].

2.4 A physical interpretation of the signs ϵ and α

It is instructive to analyse the relation between the signs ϵ and α and the charges of the D -branes and O -planes. These signs are defined in the second paragraph of the previous subsection and in appendix B.

The $D = 6$ \mathbb{Z}_2 orientifold of Gimon and Polchinski [11] contains one $O9^-$ -plane (of RR charge -32 and NSNS charge -32) and 16 $O5^-$ -planes (each of RR charge -2 and NSNS charge -2) extended in the 6 space-time dimensions and located at the 16 fixed points of ΩR in the compact dimensions. Tadpole cancellation requires the introduction of 32 $D9$ -branes and 32 $D5$ -branes (each of RR charge $+1$ and NSNS charge $+1$). This model is supersymmetric. Indeed, the RR tadpoles and the NSNS tadpoles vanish both. Looking at the specific form of the Klein bottle and cylinder amplitude, we find that $\epsilon = \alpha = 1$ in the GP model.

Antoniadis, Dudas and Sagnotti [16] and Aldazabal and Uranga [17] construct a different $D = 6$ \mathbb{Z}_2 orientifold by introducing a sign $\epsilon = -1$ in the twisted Klein bottle contribution. This corresponds to replacing the 16 $O5^-$ -planes of the GP model by 16 $O5^+$ -planes (each

of RR charge +2 and NSNS charge +2). Thus, ϵ is the relative sign between the charges of the $O9$ -planes and the $O5$ -planes [16]. To cancel the RR tadpoles, the authors of [16, 17] introduce 32 anti- $D5$ -branes (each of RR charge -1 and NSNS charge $+1$). This leads to a sign flip of the coefficient α in the cylinder contribution to the RR tadpoles (e.g. in eq. (2.19)). The corresponding contribution to the NSNS tadpoles remains unchanged. Thus, one has $\alpha_{RR} = -1$, $\alpha_{NSNS} = +1$. We deduce that α is the relative sign between the charges of $D9$ -branes and $D5$ -branes. This model is not supersymmetric. Indeed, there is a clear asymmetry between the RR tadpoles and the NSNS tadpoles. The former vanish whereas the latter lead to a non-vanishing potential for the dilaton.

The orientifold of Dabholkar and Park [12] and of Blum and Zaffaroni [13] is yet another $D = 6$ \mathbb{Z}_2 model. They also introduce a sign $\epsilon = -1$ in the twisted Klein bottle contribution. This is a consequence of the operator T in the DPBZ Ω -projection (2.1). Geometrically, this means that their model contains 16 $O5^+$ -planes. This leads to a puzzle: On the one hand the DPBZ model is clearly supersymmetric (it has an equivalent description as a supersymmetric F-theory vacuum), on the other hand there is no obvious object that could cancel the NSNS charge of the $O5^+$ -planes. The authors of [12] noticed that the alternative ground state of the 95 sector, such that $\Omega^2|_{95} = 1$, leads to a change in the twisted 95 cylinder amplitude. Thus, $\alpha = -1$ for this model. Unlike the case of the previous paragraph, this sign flip affects the RR tadpoles as well as the NSNS tadpoles. Assuming that the interpretation of α as the charge difference between $D9$ -branes and $D5$ -branes is correct, this leads us to propose the following solution to the above puzzle: The $D5$ -branes present in the DPBZ model have the peculiar property of having RR charge -1 and NSNS charge -1 . They might be called $D5^-$ -branes because their relation to the standard $D5$ -branes is the same as the relation of the $O5^-$ -planes to the $O5^+$ -planes.

The orientifold models that we present in this article are the direct generalization of the DPBZ model to $D = 4$ and orbifold groups \mathbb{Z}_N . All these models have $\epsilon = \alpha = -1$. Therefore one might think of our models as containing 16 $O5^+$ -planes and 32 $D5^-$ -branes.

3 Description of the models

All the models contain 32 $D9$ -branes and 32 $D5_3$ -branes (wrapping the third internal torus). In general there are two different orientifolds for each orbifold group Γ .

(i) The orientifold without vector structure, i.e. $\mu_9 = \mu_5 = -1$. For consistency, we need to have $\gamma_{\Omega,9}$ symmetric and $\gamma_{\Omega,5}$ antisymmetric. This is the usual GP projection, with $\epsilon = \alpha = -\Omega^2|_{95_3} = 1$.

(ii) The orientifold with vector structure, i.e. $\mu_9 = \mu_5 = 1$. For consistency, we need to have $\gamma_{\Omega,9}$ and $\gamma_{\Omega,5}$ both symmetric. This is the alternative projection of DPBZ, with $\epsilon = \alpha = -\Omega^2|_{95_3} = -1$.

It turns out that some of the models of type (i) are not consistent because they have uncanceled tadpoles. This inconsistency can also be seen as the impossibility to find a brane configuration that leads to a gauge anomaly free spectrum. (The equivalence of these two point of views has been studied in [21].) However, there exists at least one solution to the tadpole equations for each of the models of type (ii).⁶ In this section, we give the complete spectrum of all these models.

3.1 \mathbb{Z}_4 , $v = \frac{1}{4}(1, 1, -2)$

It is possible to put all the $D5_3$ -branes at the origin of the first two tori. The tadpole cancellation conditions for this configuration are:

$$\begin{aligned} k = 0 : \quad & \text{Tr}(\gamma_{0,9}) = \text{Tr}(\gamma_{0,5}) = 32 \\ k = 1 : \quad & \text{Tr}(\gamma_{1,9}) - 2 \text{Tr}(\gamma_{1,5}) = 0, \quad \text{Tr}(\gamma_{1,9}) = 0 \\ k = 2 : \quad & \text{Tr}(\gamma_{2,9}) - 4 \text{Tr}(\gamma_{2,5}) = 0, \quad \text{Tr}(\gamma_{2,9}) = 0 \end{aligned} \tag{3.1}$$

The first condition for the sectors $k = 1, 2$ corresponds to the fixed point at the origin; the second condition corresponds to the remaining fixed points, where no $D5_3$ -branes are present. All these conditions can be satisfied simultaneously. The solution is unique and leads to the spectrum displayed in table 3. The matter representations are denoted by their Young tableaux, the indices correspond to the number of the gauge group factor under which these fields transform.

This model is selfdual under T-duality in the first and second tori.

Following the argument of [21], one can see the relation between the non-Abelian anomalies and the tadpoles of the $k = 1$ sector. (The sectors $k = 0, 2$ fix the third torus and give additional constraints.) The gauge group of the \mathbb{Z}_4 orientifold with vector structure has the general form $SO(n_1) \times U(n_2) \times SO(n_3)$. The conditions on the numbers n_1, n_2, n_3 that are necessary for an anomaly free spectrum can be rewritten in terms of traces of γ matrices: $-2 \text{Tr}(\gamma_{1,9}) + \text{Tr}(\gamma_{1,5}) = 0$ (for the 99 gauge group) and $\text{Tr}(\gamma_{1,9}) - 2 \text{Tr}(\gamma_{1,5}) = 0$ (for the 55 gauge group). These two equations are equivalent to the two tadpole equations of the $k = 1$ sector.

⁶The case of the \mathbb{Z}'_8 orientifold has to be revisited, see below.

$\mathbb{Z}_4, v = \frac{1}{4}(1, 1, -2), \mu = 1$	
open string spectrum	
sector	gauge group / matter fields
99	$SO(8)_1 \times U(8)_2 \times SO(8)_3$ $2(\square_1, \bar{\square}_2), 2(\square_2, \square_3), (\square_1, \square_3), \bar{\square}_2, \bar{\square}_2$
55	identical to 99
95	$(\square_1, \square_2), (\square_2, \square_1), (\bar{\square}_2, \square_3)(\square_3, \bar{\square}_2)$
closed string spectrum	
sector	$\mathcal{N} = 1$ multiplets
untw.	gravity, 1 lin., 6 chir.
$k = N/2$	10 lin., 6 vec.
$0 < k < N/2$	16 lin.

Table 3: Spectrum of the \mathbf{Z}_4 orientifold with vector structure.

Notice the similarity of this model with the non-supersymmetric \mathbb{Z}_4 orientifold of [19]. The difference is that the authors of [19] use the standard Ω action of GP, $\Omega^2|_{95_3} = -1$, which requires 32 anti- $D5_3$ -branes to cancel the untwisted tadpoles. For consistency, $\gamma_{\Omega,5}$ has to be antisymmetric. According to the general formula (2.7) this gives symplectic instead of orthogonal gauge group factors. Therefore, the gauge group of the 55 sector of their model is $USp(8) \times U(8) \times USp(8)$.

3.2 $\mathbf{Z}_6, v = \frac{1}{6}(1, 1, -2)$

It is possible to put all the $D5_3$ -branes at the origin of the first two tori. The tadpole cancellation conditions for this configuration are:

$$\begin{aligned}
k = 0 : \quad & \text{Tr}(\gamma_{0,9}) = \text{Tr}(\gamma_{0,5}) = 32 \\
k = 1 : \quad & \text{Tr}(\gamma_{1,9}) - \text{Tr}(\gamma_{1,5}) = 0, \quad \text{Tr}(\gamma_{1,9}) = 0 \\
k = 2 : \quad & \text{Tr}(\gamma_{2,9}) - 3 \text{Tr}(\gamma_{2,5}) = 8, \quad \text{Tr}(\gamma_{2,9}) = -4 \\
k = 3 : \quad & \text{Tr}(\gamma_{3,9}) - 4 \text{Tr}(\gamma_{3,5}) = 0, \quad \text{Tr}(\gamma_{3,9}) = 0
\end{aligned} \tag{3.2}$$

Again, the first condition for each twisted sector corresponds to the fixed point at the origin; the second condition corresponds to the remaining fixed points, where no $D5_3$ -branes are

present. In the $k = 2$ sector, there are 9 fixed points in the first two tori. The untwisted Klein bottle contribution $\mathcal{K}_0(1)$ is only present at the origin (this is the only fixed point under $k' = k/2 = 1$). The twisted Klein bottle contribution $\mathcal{K}_1(1)$ is present at all of the 9 fixed points (all of them are fixed under $k' = k/2 + N/2 = 4$). This explains the right hand side of the $k = 2$ tadpole conditions. There are five solutions to these equations. The gauge group and the matter are shown in table 4.

$\mathbb{Z}_6, v = \frac{1}{6}(1, 1, -2), \mu = 1$	
open string spectrum	
sector	gauge group / matter fields
99	$SO(8 - 2n)_1 \times U(8 - n)_2 \times U(4 + n)_3 \times SO(2n)_4$ $2(\square_1, \bar{\square}_2), 2(\square_2, \bar{\square}_3), 2(\square_3, \square_4), (\square_1, \square_3), (\bar{\square}_2, \square_4), \bar{\square}_2, \bar{\square}_3$
55	identical to 99
95	$(\square_1, \square_2), (\square_2, \square_1), (\bar{\square}_2, \square_3), (\square_3, \bar{\square}_2), (\bar{\square}_3, \square_4), (\square_4, \bar{\square}_3)$
closed string spectrum	
sector	$\mathcal{N} = 1$ multiplets
untw.	gravity, 1 lin., 5 chir.
$k = N/2$	6 lin., 5 vec.
$0 < k < N/2$	18 lin.

Table 4: Spectrum of the \mathbb{Z}_6 orientifold with vector structure. The parameter n can take the five values $n = 0, \dots, 4$.

All the five models are selfdual under T-duality in the first two tori. One can verify that the conditions for absence of non-Abelian gauge anomalies coincide with the tadpole conditions for the sectors $k = 1, 2$.

For completeness and for comparison, we also give the spectrum of the \mathbb{Z}_6 orientifold without vector structure, table 5. This orientifold has been analysed by the authors of [4, 6, 21]. There are 13 consistent models with all the $D5_3$ -branes at the origin. One of them is selfdual under T-duality in the first two tori. The remaining 12 are organized in pairs of T-dual models.

$\mathbb{Z}_6, v = \frac{1}{6}(1, 1, -2), \mu = -1$	
open string spectrum	
sector	gauge group / matter fields
99	$U(n)_1 \times U(4)_2 \times U(12-n)_3$ $2(\square_1, \bar{\square}_2), 2(\square_2, \bar{\square}_3), (\square_1, \square_2), (\bar{\square}_2, \bar{\square}_3), (\square_3, \bar{\square}_1), 2\bar{\square}_1, 2\square_3$
55	$U(12-n) \times U(4) \times U(n)$ matter identical to 99
95	$(\bar{\square}_1, \bar{\square}_1), (\square_1, \bar{\square}_2), (\bar{\square}_2, \square_1), (\square_2, \bar{\square}_3), (\bar{\square}_3, \square_2), (\square_3, \square_3)$
closed string spectrum	
sector	$\mathcal{N} = 1$ multiplets
untw.	gravity, 1 lin., 5 chir.
$k = N/2$	11 chir.
$0 < k < N/2$	18 lin.

Table 5: Spectrum of the \mathbf{Z}_6 orientifold without vector structure. The parameter n can take the 13 values $n = 0, \dots, 12$.

3.3 $\mathbf{Z}'_6, v = \frac{1}{6}(1, -3, 2)$

It is possible to put all the $D5_3$ -branes at the origin of the first two tori. The tadpole cancellation conditions for this configuration are:

$$\begin{aligned}
k = 0 : \quad & \text{Tr}(\gamma_{0,9}) = \text{Tr}(\gamma_{0,5}) = 32 \\
k = 1 : \quad & \text{Tr}(\gamma_{1,9}) + 2 \text{Tr}(\gamma_{1,5}) = 0, \quad \text{Tr}(\gamma_{1,9}) = 0 \\
k = 2 : \quad & \text{Tr}(\gamma_{2,9}) = 8, \quad \text{Tr}(\gamma_{2,5}) = 8 \\
k = 3 : \quad & \text{Tr}(\gamma_{3,9}) - 4 \text{Tr}(\gamma_{3,5}) = 0, \quad \text{Tr}(\gamma_{3,9}) = 0
\end{aligned} \tag{3.3}$$

There is a unique solution to these equations, leading to the spectrum shown in table 6. This model is selfdual under T-duality in the first two tori. As expected, the conditions for non-Abelian anomaly freedom are equivalent to the tadpole conditions for the sectors $k = 1, 2$.

For completeness and for comparison, we also give the spectrum of the \mathbf{Z}'_6 orientifold without vector structure and all $D5_3$ -branes at the origin, table 7. This orientifold has been

$\mathbb{Z}'_6, v = \frac{1}{6}(1, -3, 2), \mu = 1$	
open string spectrum	
sector	gauge group / matter fields
99	$SO(8)_1 \times U(4)_2 \times U(4)_3 \times SO(8)_4$ $(\square_1, \bar{\square}_2), (\square_1, \bar{\square}_3), (\square_1, \square_4), (\square_2, \square_3), (\square_2, \bar{\square}_3), (\bar{\square}_2, \bar{\square}_3)$ $(\square_2, \square_4), (\square_3, \square_4), \bar{\square}_3, \bar{\bar{\square}}_2$
55	identical to 99
95	$(\square_1, \bar{\square}_2), (\bar{\square}_2, \square_1), (\square_2, \bar{\square}_3), (\bar{\square}_3, \square_2), (\square_3, \square_4), (\square_4, \square_3)$
closed string spectrum	
sector	$\mathcal{N} = 1$ multiplets
untw.	gravity, 1 lin., 4 chir.
$k = N/2$	8 lin., 4 vec.
$0 < k < N/2$	18 lin., 9 chir., 3 vec.

Table 6: Spectrum of the \mathbb{Z}'_6 orientifold with vector structure.

analysed by the authors of [5, 6, 21]. The solution to the tadpole equations is unique and selfdual under T-duality in the first two tori.

3.4 $\mathbb{Z}_8, v = \frac{1}{8}(1, 3, -4)$

The construction of this orientifold model is more complicated. We will see that it is not possible to satisfy the tadpole conditions if all $D5_3$ -branes are put at the origin. Let us first analyse the fixed points of the twisted sectors. The lattice corresponding to the first two complex planes cannot be decomposed in two two-dimensional sublattices.⁷ Nevertheless, the Lefschetz formula for the number of fixed points is still valid, when the above shift vector is used. One finds two \mathbb{Z}_8 fixed points in the first two complex planes, one of them is the origin. In the $k = 2$ sector there are four \mathbb{Z}_4 fixed points (more precisely, fixed tori extended in the third complex plane). Two of them are also fixed under \mathbb{Z}_8 , the other two are permuted under the action of \mathbb{Z}_8 . The $k = 3$ sector only has the two \mathbb{Z}_8 fixed points and in the $k = 4$ sector we find 16 \mathbb{Z}_2 fixed points.

We want to find the simplest brane configuration that satisfies all the tadpole conditions.

⁷The compactification lattice of the \mathbb{Z}_8 orientifold is the root lattice of the Lie algebra $B_4 \times D_2$.

$\mathbb{Z}'_6, v = \frac{1}{6}(1, -3, 2), \mu = -1$	
open string spectrum	
sector	gauge group / matter fields
99	$U(4)_1 \times U(8)_2 \times U(4)_3$ $(\square_1, \bar{\square}_2), (\bar{\square}_1, \bar{\square}_2), (\square_1, \square_3), (\square_1, \bar{\square}_3), (\bar{\square}_1, \bar{\square}_3),$ $(\square_2, \square_3), (\square_2, \bar{\square}_3), \bar{\square}_1, \bar{\square}_2, \bar{\square}_2, \bar{\square}_3$
55	identical to 99
95	$(\square_1, \square_1), (\bar{\square}_1, \square_2), (\square_2, \bar{\square}_1), (\bar{\square}_2, \square_3), (\square_3, \bar{\square}_2), (\bar{\square}_3, \bar{\square}_3)$
closed string spectrum	
sector	$\mathcal{N} = 1$ multiplets
untw.	gravity, 1 lin., 5 chir.
$k = N/2$	12 chir.
$0 < k < N/2$	18 lin., 9 chir., 3 vec.

Table 7: Spectrum of the \mathbb{Z}'_6 orientifold without vector structure.

The 32 $D5_3$ -branes are divided into several sets. Each of these sets of branes is located at a different point in the first two complex planes. We denote the two \mathbb{Z}_8 fixed points by the index $i = 0, 1$, the two \mathbb{Z}_4 fixed points which are not fixed under \mathbb{Z}_8 by the index $J = 2, 3$ and the 12 \mathbb{Z}_2 fixed points which are not fixed under \mathbb{Z}_4 by the index $I = 4, \dots, 15$. Then the tadpole cancellation conditions are:

$$\begin{aligned}
k = 0 : \quad & \text{Tr}(\gamma_{0,9}) = \sum_i \text{Tr}(\gamma_{0,5,i}) + \sum_J \text{Tr}(\gamma_{0,5,J}) + \sum_I \text{Tr}(\gamma_{0,5,I}) = 32 \\
k = 1 : \quad & \text{Tr}(\gamma_{1,9}) - \sqrt{2} \text{Tr}(\gamma_{1,5,i}) = 0 \\
k = 2 : \quad & \text{Tr}(\gamma_{2,9}) - 2 \text{Tr}(\gamma_{2,5,i}) = 0, \quad \text{Tr}(\gamma_{2,9}) - 2 \text{Tr}(\gamma_{2,5,J}) = 0 \\
k = 3 : \quad & \text{Tr}(\gamma_{3,9}) + \sqrt{2} \text{Tr}(\gamma_{3,5,i}) = 0 \\
k = 4 : \quad & \text{Tr}(\gamma_{4,9}) + 4 \text{Tr}(\gamma_{4,5,i}) = 32, \quad \text{Tr}(\gamma_{4,9}) + 4 \text{Tr}(\gamma_{4,5,J}) = 32, \\
& \text{Tr}(\gamma_{4,9}) + 4 \text{Tr}(\gamma_{4,5,I}) = 0
\end{aligned} \tag{3.4}$$

Note that there is a non-vanishing contribution of the Klein bottle to the four \mathbb{Z}_4 fixed points in the $k = 4$ sector. The contribution of $\mathcal{K}_0(2)$ and $\mathcal{K}_1(2)$ is not present at the other 12 \mathbb{Z}_2 fixed points because the twists $k' = k/2 = 2$ and $k' = k/2 + N/2 = 6$ only leave the \mathbb{Z}_4 fixed

points invariant. This in contrast to the models without vector structure, where the Klein bottle contribution to the $k = N/2$ sector always vanishes at all fixed points.

To cancel the tadpoles from the $k = 4$ sector, some branes need to be located at the two \mathbb{Z}_8 fixed points $i = 0, 1$ and at the pair of \mathbb{Z}_4 fixed points $J = 2, 3$ (the branes at the two points $J = 2, 3$ must be identical to be \mathbb{Z}_8 -invariant). The tadpole equations have many solutions. The most symmetric one corresponds to a configuration with 8 $D5_3$ -branes at each of the four \mathbb{Z}_4 fixed points. There are still 25 different solutions for the γ matrices. The gauge groups corresponding to these solutions depend on the parameters $m = 0, \dots, 4$, $n = 0, \dots, 8$, satisfying the conditions $m + n \leq 8$ and $m \leq n$. In table 8, we give the complete spectrum. We denoted the different sets of $D5_3$ -branes by $5_0, 5_1, 5_J$, referring to the fixed point at the origin, the second \mathbb{Z}_8 fixed point and the pair of \mathbb{Z}_4 fixed points which are permuted under the action of \mathbb{Z}_8 . Note that the spectrum of the $5_J 5_J$ sector is that of an \mathbb{Z}_4 orientifold with shift vector $v = \frac{1}{4}(1, 3, -4)$.

The tadpole conditions of the sectors $k = 1, 3$ are equivalent the conditions that can be derived from the requirement of anomaly freedom. Note that the tadpole equations of the $k = 4$ sector forced us to put $D5_3$ -branes at different fixed points. However, these conditions are not necessary for anomaly cancellation. Indeed, it would be possible to construct an anomaly free \mathbb{Z}_8 orientifold with all $D5_3$ -branes sitting at the origin. In the non-compact limit, such a model is consistent.

3.5 $\mathbb{Z}'_8, v = \frac{1}{8}(1, -3, 2)$

We did not find a consistent solution for the \mathbb{Z}'_8 orientifold. Below we show the tadpole cancellation conditions and the general spectrum of D -branes at a \mathbb{Z}'_8 singularity. The fixed points in the first two complex planes are identical to the fixed points of the \mathbb{Z}_8 model.⁸ The only difference is that now the third complex plane is only fixed under the $k = 4$ twist. There are two \mathbb{Z}_8 fixed points in the first two complex planes, one of them is the origin. In the $k = 2$ sector there are four \mathbb{Z}_4 fixed points. Two of them are also fixed under \mathbb{Z}_8 , the other two are permuted under the action of \mathbb{Z}_8 . The $k = 3$ sector only has the two \mathbb{Z}_8 fixed points and in the $k = 4$ sector we find 16 \mathbb{Z}_2 fixed points (more precisely, fixed tori extended in the third complex plane). As for the previous model, we denote the two \mathbb{Z}_8 fixed points by the index $i = 0, 1$, the two \mathbb{Z}_4 fixed points which are not fixed under \mathbb{Z}_8 by the index $J = 2, 3$ and the 12 \mathbb{Z}_2 fixed points which are not fixed under \mathbb{Z}_4 by the index $I = 4, \dots, 15$.

⁸The compactification lattice of the \mathbb{Z}'_8 orientifold is the root lattice of the Lie algebra $B_4 \times B_2$, i.e. the four-dimensional sublattice corresponding to the first two complex planes is not factorizable. However, the Lefschetz formula for the number of fixed points is still valid.

$\mathbb{Z}_8, v = \frac{1}{8}(1, 3, -4), \mu = 1$	
open string spectrum	
sector	gauge group / matter fields
99	$SO(8 - 2m)_1 \times U(8 - n)_2 \times U(2m)_3 \times U(n)_4 \times SO(8 - 2m)_5$ $(\square_1 \bar{\square}_2), (\square_1, \bar{\square}_4), (\square_1, \square_5), (\square_2, \bar{\square}_3), (\bar{\square}_2, \bar{\square}_3), (\square_2, \square_4)$ $(\bar{\square}_2, \bar{\square}_4), (\square_2, \square_5), (\square_3, \square_4), (\square_3, \bar{\square}_4), (\square_4, \square_5), \bar{\square}_3, \bar{\bar{\square}}_3$
$5_0 5_0$	$SO(8 - m - n)_1 \times U(m)_3 \times SO(n - m)_5$ $(\square_1, \square_5), \bar{\square}_3, \bar{\bar{\square}}_3$
$5_1 5_1$	identical to $5_0 5_0$
$5_J 5_J$	$SO(8 - 2m)_1 \times SO(2m)_3$ $\bar{\square}_1, \bar{\bar{\square}}_3$
95_0	$(\square_1, \square_3), (\square_2, \square_2), (\bar{\square}_2, \square_4), (\square_3, \square_1), (\bar{\square}_3, \square_5), (\square_4, \bar{\square}_2)$ $(\bar{\square}_4, \bar{\square}_4), (\square_5, \bar{\square}_3)$
95_1	identical to 95_0
95_J	$(\square_1, \square_3), (\square_3, \square_1), (\bar{\square}_3, \square_1), (\square_5, \square_3)$
closed string spectrum	
sector	$\mathcal{N} = 1$ multiplets
untw.	gravity, 1 lin., 4 chir.
$k = N/2$	6 lin., 4 vec.
$0 < k < N/2$	19 lin., 4 chir., 1 vec.

Table 8: Spectrum of the \mathbf{Z}_8 orientifold with vector structure. The parameters $m = 0, \dots, 4$, $n = 0, \dots, 8$ satisfy $m + n \leq 8$ and $m \leq n$. Some of the possible gauge group factors are missing (e.g. the second and fourth factor in $5_i 5_i$ sectors). This is due to the tadpole cancellation conditions which force the rank of these group factors to vanish.

Then the tadpole cancellation conditions are:

$$\begin{aligned}
k = 0 : \quad & \text{Tr}(\gamma_{0,9}) = \sum_i \text{Tr}(\gamma_{0,5,i}) + \sum_J \text{Tr}(\gamma_{0,5,J}) + \sum_I \text{Tr}(\gamma_{0,5,I}) = 32 \\
k = 1 : \quad & \text{Tr}(\gamma_{1,9}) + \sqrt{2} \text{Tr}(\gamma_{1,5,i}) = 0 \\
k = 2 : \quad & \text{Tr}(\gamma_{2,9}) + 2 \text{Tr}(\gamma_{2,5,i}) = 16, \quad \text{Tr}(\gamma_{2,9}) + 2 \text{Tr}(\gamma_{2,5,J}) = 0 \\
k = 3 : \quad & \text{Tr}(\gamma_{3,9}) - \sqrt{2} \text{Tr}(\gamma_{3,5,i}) = 0 \\
k = 4 : \quad & \text{Tr}(\gamma_{4,9}) - 4 \text{Tr}(\gamma_{4,5,i}) = 0, \quad \text{Tr}(\gamma_{4,9}) - 4 \text{Tr}(\gamma_{4,5,J}) = 0, \\
& \text{Tr}(\gamma_{4,9}) - 4 \text{Tr}(\gamma_{4,5,I}) = 0
\end{aligned} \tag{3.5}$$

From the first condition of the $k = 2$ twisted sector, we can see that some $D5_3$ -branes have to sit at each of the two \mathbb{Z}_8 fixed points to cancel the tadpoles. This is the only orientifold where the necessity to put $D5_3$ -branes at different fixed points arises due to the tadpole conditions of a sector without fixed planes. Equivalently it can be verified that the general spectrum of a configuration with all $D5_3$ -branes at the origin has no solution for the γ matrices such that the spectrum is free of non-Abelian gauge anomalies.

The general spectrum for a configuration with two sets of $D5_3$ -branes located at the two \mathbb{Z}_8 fixed points $i = 0, 1$, is shown in table 9. The numbers n_j , m_j , l_j should be fixed by the tadpole conditions. We must have overlooked some subtlety because it turns out that the conditions for anomaly freedom of $U(n_2)$, $U(n_4)$, $U(m_2)$, $U(m_4)$, $U(l_2)$ and $U(l_4)$ lead to $\text{Tr}(\gamma_{2,9}) = 0$, $\text{Tr}(\gamma_{2,5,i}) = 4$. This is incompatible with the tadpole equations of the $k = 2$ sector.

3.6 \mathbf{Z}_{12} , $v = \frac{1}{12}(1, -5, 4)$

It is not possible to locate all the $D5_3$ -branes at the origin. In this case, the tadpole cancellation conditions of the order-two sector $k = 6$ are responsible for this (i.e. it is not related to non-Abelian anomaly cancellation). This is analogous to the case of the \mathbb{Z}_8 orientifold. In the sectors $k = 1, 2, 5$ there is only fixed point in the first two complex planes⁹: the origin. In the $k = 3$ sector there are four \mathbb{Z}_4 fixed points (more precisely, four fixed tori extended in the third complex plane). In the $k = 4$ sector there are 9 \mathbb{Z}_3 fixed points in the first two complex planes and in the $k = 6$ sector 16 \mathbb{Z}_2 fixed points. We denote the origin by 0, the three \mathbb{Z}_4 fixed points which are not fixed under \mathbf{Z}_{12} by the index $i = 1, 2, 3$,

⁹The compactification lattice of the \mathbf{Z}_{12} orientifold is the root lattice of the Lie algebra $F_4 \times A_2$, i.e. the four-dimensional sublattice corresponding to the first two complex planes is not factorizable. However, the Lefschetz formula for the number of fixed points is still valid.

$\mathbb{Z}'_8, v = \frac{1}{8}(1, -3, 2), \mu = 1$	
open string spectrum	
sector	gauge group / matter fields
99	$SO(n_1) \times U(n_2) \times U(n_3) \times U(n_4) \times SO(n_5)$ $(\square_1, \bar{\square}_2), (\square_1, \bar{\square}_3), (\square_1, \square_4), (\square_2, \square_3), (\square_2, \bar{\square}_3), (\square_2, \bar{\square}_4), (\bar{\square}_2, \square_5)$ $(\square_3, \bar{\square}_4), (\bar{\square}_3, \bar{\square}_4), (\square_3, \square_5), (\square_4, \square_5), \bar{\square}_2, \square_4$
$5_0 5_0$	$SO(m_1) \times U(m_2) \times U(m_3) \times U(m_4) \times SO(m_5)$ matter identical to 99
$5_1 5_1$	$SO(l_1) \times U(l_2) \times U(l_3) \times U(l_4) \times SO(l_5)$ matter identical to 99
95_0	$(\square_1, \bar{\square}_2), (\bar{\square}_2, \square_1), (\square_2, \bar{\square}_3), (\bar{\square}_3, \square_2), (\square_3, \bar{\square}_4), (\bar{\square}_4, \square_3)$ $(\square_4, \square_5), (\square_5, \square_4)$
95_1	identical to 95_0
closed string spectrum	
sector	$\mathcal{N} = 1$ multiplets
untw.	gravity, 1 lin., 3 chir.
$k = N/2$	6 lin., 3 vec.
$0 < k < N/2$	18 lin.

Table 9: Spectrum of the \mathbf{Z}'_8 orientifold with vector structure. The parameters n_j, m_j, l_j should be fixed by solving the tadpole cancellation conditions.

the 8 \mathbb{Z}_3 fixed points which are not fixed under \mathbb{Z}_{12} by the index $J = 1, \dots, 8$ and the 12 \mathbb{Z}_2 fixed points which are not fixed under \mathbb{Z}_4 by the index $I = 4, \dots, 15$. Then the tadpole cancellation conditions are:

$$\begin{aligned}
k = 0 : \quad & \text{Tr}(\gamma_{0,9}) = \text{Tr}(\gamma_{0,5,0}) + \sum_i \text{Tr}(\gamma_{0,5,i}) + \sum_J \text{Tr}(\gamma_{0,5,J}) + \sum_I \text{Tr}(\gamma_{0,5,I}) = 32 \\
k = 1 : \quad & \text{Tr}(\gamma_{1,9}) + \text{Tr}(\gamma_{1,5,0}) = 0 \\
k = 2 : \quad & \text{Tr}(\gamma_{2,9}) + \text{Tr}(\gamma_{2,5,0}) = 8 \\
k = 3 : \quad & \text{Tr}(\gamma_{3,9}) - 2 \text{Tr}(\gamma_{3,5,0}) = 0, \quad \text{Tr}(\gamma_{3,9}) - 2 \text{Tr}(\gamma_{3,5,i}) = 0 \\
k = 4 : \quad & \text{Tr}(\gamma_{4,9}) - 3 \text{Tr}(\gamma_{4,5,0}) = 8, \quad \text{Tr}(\gamma_{4,9}) - 3 \text{Tr}(\gamma_{4,5,J}) = -4 \\
k = 5 : \quad & \text{Tr}(\gamma_{5,9}) + \text{Tr}(\gamma_{5,5,0}) = 0 \\
k = 6 : \quad & \text{Tr}(\gamma_{6,9}) + 4 \text{Tr} \gamma_{6,5,0} = 32, \quad \text{Tr}(\gamma_{6,9}) + 4 \text{Tr} \gamma_{6,5,i} = 32, \\
& \text{Tr}(\gamma_{6,9}) + 4 \text{Tr} \gamma_{6,5,I} = 0
\end{aligned} \tag{3.6}$$

In order to cancel the $k = 6$ twisted tadpoles there must be some $D5_3$ -branes at each \mathbb{Z}_4 fixed point. One of these is the origin and the other three are permuted by the \mathbb{Z}_{12} action. Thus the branes at the three points $i = 1, 2, 3$ must be identical to be \mathbb{Z}_{12} -invariant. The tadpole equations have many solutions. The most symmetric one corresponds to a configuration with 8 $D5_3$ -branes at each of the four \mathbb{Z}_4 fixed points. There are still 5 different solutions for the γ matrices. In table 10, we give the complete spectrum. We denoted the different sets of $D5_3$ -branes by $5_0, 5_i$, referring to the fixed point at the origin and the three \mathbb{Z}_4 fixed points which are permuted under the action under the action of \mathbb{Z}_{12} . Note that the spectrum of the $5_i 5_i$ sector is that of an \mathbb{Z}_4 orientifold with shift vector $v = \frac{1}{4}(1, 5, -4)$.

We verified that the requirement of anomaly cancellation is equivalent to the twisted tadpole equations from the sectors $k = 1, 2, 4, 5$. The tadpole conditions from the $k = 6$ sector, which require to distribute the $D5_3$ -branes over different fixed points, are not necessary for anomaly cancellation. One could, in principle, put all the $D5_3$ -branes at the origin and produce a model free of anomalies.

These models are not selfdual under T-duality because the $D5_3$ -branes are at several fixed points. The dual model will require Wilson lines in the $D9$ -brane sector.

The orientifold without vector structure has been constructed in [6, 21]. There are 235 models without vector structure if all the $D5_3$ -branes sit at the origin. For completeness and for comparison we display the spectrum in table 11.

$\mathbb{Z}_{12}, v = \frac{1}{12}(1, -5, 4), \mu = 1$	
open string spectrum	
sector	gauge group / matter fields
99	$SO(2n)_1 \times U(4)_2 \times U(4-n)_3 \times U(n)_5 \times U(4)_6 \times SO(8-2n)_7$ $(\square_1, \bar{\square}_2), (\square_1, \bar{\square}_5), (\square_1, \square_6), (\square_2, \bar{\square}_3), (\square_2, \square_5), (\square_2, \bar{\square}_6)$ $(\bar{\square}_2, \square_7), (\bar{\square}_3, \bar{\square}_6), (\square_3, \square_7), (\square_5, \bar{\square}_6), (\square_6, \square_7), \bar{\square}_3, \square_5$
$5_0 5_0$	$U(4-n)_3 \times U(n)_5$ $\bar{\square}_3, \square_5$
$5_i 5_i$	$SO(2n)_1 \times SO(8-2n)_3$ \square_1, \square_3
95_0	$(\square_1, \bar{\square}_3), (\square_3, \bar{\square}_5), (\bar{\square}_5, \square_3), (\square_7, \square_5)$
95_i	$(\square_7, \square_1), (\square_1, \square_3)$
closed string spectrum	
sector	$\mathcal{N} = 1$ multiplets
untw.	gravity, 1 lin., 3 chir.
$k = N/2$	4 lin., 3 vec.
$0 < k < N/2$	20 lin., 3 chir., 1 vec.

Table 10: Spectrum of the \mathbf{Z}_{12} orientifold with vector structure. There are 5 solutions parametrized by $n = 0, \dots, 4$. The difference between the 99 and the $5_0 5_0$ sector is due to the tadpole cancellation conditions which force many of the possible group factors to vanish.

$\mathbb{Z}_{12}, v = \frac{1}{12}(1, -5, 4), \mu = -1$	
open string spectrum	
sector	gauge group / matter fields
99	$U(l)_1 \times U(m)_2 \times U(n)_3 \times U(4 + m - l)_4 \times U(4 - m)_5 \times U(8 - m - n)_6$ $(\square_1, \bar{\square}_2), (\bar{\square}_1, \bar{\square}_4), (\square_1, \square_5), (\square_1, \bar{\square}_5), (\square_2, \bar{\square}_3), (\bar{\square}_2, \bar{\square}_3)$ $(\square_2, \square_4), (\square_2, \bar{\square}_6), (\bar{\square}_2, \bar{\square}_6), (\square_3, \bar{\square}_4), (\bar{\square}_3, \bar{\square}_5), (\square_3, \square_6)$ $(\square_4, \square_5), (\square_4, \bar{\square}_5), (\square_5, \bar{\square}_6), (\square_6, \bar{\square}_1), \bar{\square}_1, \square_3 \bar{\square}_4, \square_6$
55	identical to 99
95	$(\square_1, \square_2), (\bar{\square}_1, \square_3), (\square_2, \square_1), (\bar{\square}_2, \square_4), (\square_3, \bar{\square}_1), (\bar{\square}_3, \square_5)$ $(\square_4, \bar{\square}_2), (\bar{\square}_4, \square_6), (\square_5, \bar{\square}_3), (\bar{\square}_5, \bar{\square}_6), (\square_6, \bar{\square}_4), (\bar{\square}_6, \bar{\square}_5)$
closed string spectrum	
sector	$\mathcal{N} = 1$ multiplets
untw.	gravity, 1 lin., 3 chir.
$k = N/2$	7 chir.
$0 < k < N/2$	20 lin., 3 chir., 1 vec.

Table 11: Spectrum of the \mathbf{Z}_{12} orientifold without vector structure. There are 235 solutions parametrized by $l, n = 0, \dots, 8, m = 0, \dots, 4$, satisfying $l - m \leq 4$ and $n + m \leq 8$.

3.7 \mathbf{Z}'_{12} , $v = \frac{1}{12}(1, 5, -6)$

This orientifold can be consistently constructed with all $D5_3$ -branes sitting at the origin of the first two complex planes.¹⁰ In the $k = 1, 2, 5$ sectors, only the origin is fixed. In the sector $k = 3$, there are four fixed tori, one of them is \mathbb{Z}_{12} invariant (the origin) and the other form a triplet of \mathbb{Z}_4 fixed points permuted by the \mathbb{Z}_{12} element. The $k = 4$ twisted sector has 9 fixed points in the first two complex planes. The tadpole cancellation conditions for all $D5_3$ -branes at the origin are:

$$\begin{aligned}
k = 0 : \quad & \text{Tr}(\gamma_{0,9}) = \text{Tr}(\gamma_{0,5}) = 32 \\
k = 1 : \quad & \text{Tr}(\gamma_{1,9}) - \text{Tr}(\gamma_{1,5}) = 0 \\
k = 2 : \quad & \text{Tr}(\gamma_{2,9}) - \text{Tr}(\gamma_{2,5}) = 0 \\
k = 3 : \quad & \text{Tr}(\gamma_{3,9}) + 2 \text{Tr}(\gamma_{3,5}) = 0, \quad \text{Tr}(\gamma_{3,9}) = 0 \\
k = 4 : \quad & \text{Tr}(\gamma_{4,9}) + 3 \text{Tr}(\gamma_{4,5}) = 32, \quad \text{Tr}(\gamma_{4,9}) = 8 \\
k = 5 : \quad & \text{Tr}(\gamma_{5,9}) - \text{Tr}(\gamma_{5,5}) = 0 \\
k = 6 : \quad & \text{Tr}(\gamma_{6,9}) - 4 \text{Tr}(\gamma_{6,5}) = 0, \quad \text{Tr}(\gamma_{6,9}) = 0
\end{aligned} \tag{3.7}$$

There are 125 solutions to the tadpole cancellation conditions, table 12. All of them are selfdual under T-duality.

We verified that the requirement of anomaly cancellation is equivalent to the twisted tadpole conditions of the sectors $k = 1, 3, 5$.

4 Conclusions

We have constructed $D = 4$, $\mathcal{N} = 1$ orientifolds with vector structure corresponding to orbifold groups \mathbb{Z}_N , N even. We found a consistent solution to the tadpole equations for each orbifold group (except \mathbb{Z}'_8) that leads to a $D = 4$, $\mathcal{N} = 1$ orbifold of the heterotic string. Due to the fact that the γ -matrices represent the action of \mathbb{Z}_N on the Chan-Paton indices only projectively, there may appear various signs in the Klein bottle, Möbius strip and cylinder amplitude. We carefully included all these signs in the tadpole computation and found new solutions. In general, there are two non-equivalent \mathbb{Z}_N orientifolds with vector structure for each even N : a non-supersymmetric model similar to the \mathbb{Z}_4 constructed in [16]

¹⁰The compactification lattice of the \mathbf{Z}'_{12} orientifold is the root lattice of the Lie algebra $F_4 \times D_2$, i.e. the four-dimensional sublattice corresponding to the first two complex planes is not factorizable. However, the Lefschetz formula for the number of fixed points is still valid.

$\mathbb{Z}'_{12}, v = \frac{1}{12}(1, 5, -6), \mu = 1$	
open string spectrum	
sector	gauge group / matter fields
99	$SO(8 + 2m - 2n)_1 \times U(l)_2 \times U(m)_3 \times U(n)_4$ $\times U(n - m)_5 \times U(8 - n - l)_6 \times SO(8 - 2m)_7$ $(\square_1, \bar{\square}_2), (\square_1, \bar{\square}_6), (\square_1, \square_7), (\square_2, \bar{\square}_3), (\bar{\square}_2, \bar{\square}_5), (\square_2, \square_6)$ $(\bar{\square}_2, \bar{\square}_6), (\square_2, \square_7), (\square_3, \bar{\square}_4), (\bar{\square}_3, \bar{\square}_4), (\square_3, \square_5), (\bar{\square}_3, \bar{\square}_5)$ $(\square_3, \square_6), (\square_4, \square_5), (\square_4, \bar{\square}_5), (\square_5, \bar{\square}_6), (\square_6, \square_7), \bar{\square}_4, \bar{\square}_4$
55	identical to 99
95	$(\square_1, \square_4), (\square_2, \square_3), (\bar{\square}_2, \square_5), (\square_3, \square_2), (\bar{\square}_3, \square_6), (\square_4, \square_1)$ $(\bar{\square}_4, \square_7), (\square_5, \bar{\square}_2), (\bar{\square}_5, \bar{\square}_6), (\square_6, \bar{\square}_3), (\bar{\square}_6, \bar{\square}_5), (\square_7, \square_4)$
closed string spectrum	
sector	$\mathcal{N} = 1$ multiplets
untw.	gravity, 1 lin., 4 chir.
$k = N/2$	4 lin., 2 vec.
$0 < k < N/2$	20 lin., 6 chir., 2 vec.

Table 12: Spectrum of the \mathbf{Z}'_{12} orientifold with vector structure. There are 125 solutions parametrized by $l, n = 0, \dots, 8, m = 0, \dots, 4$, satisfying $0 \leq n - m \leq 4$ and $l + n \leq 8$.

and a supersymmetric model constructed in the present article. For some orbifold groups, there are also consistent models without vector structure. Supersymmetric \mathbb{Z}_N orientifolds without vector structure have been constructed in [4, 6]. But their non-supersymmetric analogues should also exist.

The standard construction of \mathbb{Z}_N orientifolds (generalizations of the GP model to $D = 4$) leads to models without vector structure. They contain one $O9^-$ -plane, 16 $O5^-$ -planes, 32 $D9$ -branes and 32 $D5$ -branes. It turns out that models with vector structure are only possible if a sign in the twisted Klein bottle contribution is flipped [16, 17, 19]. This can be interpreted as replacing the $O5^-$ -planes by $O5^+$ -planes. One possibility to cancel the tadpoles of this new models is to introduce anti- $D5$ -branes. This leads to non-supersymmetric orientifolds. Another possibility consists in flipping a second sign: the action of Ω^2 on the open strings of the 95 sector [12]. This implies taking $\gamma_{\Omega,5}$ symmetric (in the standard GP construction it is antisymmetric). Furthermore, it leads to the same sign flip in the 95 cylinder contribution to the RR tadpoles as anti- $D5$ -branes do. But it does not break supersymmetry. We have not yet fully understood the physical interpretation of this second sign flip. But it is tempting to believe that it corresponds to replacing $D5$ -branes by $D5^-$ -branes which have negative RR charge and negative NSNS charge.

Quiver diagrams are a very useful tool to compute orientifold spectra. Based on the work of [26, 8], we give general rules how to obtain the orientifold quiver for a given discrete group. It is straightforward to implement this algorithm in a computer algebra program. This enables us to compute orientifold spectra in a fast and efficient way.

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Appendix

A Orientifold quivers

In this appendix, we briefly review quiver diagrams as a tool to determine the spectrum of type II orbifolds [26] and then explain how this is generalized to orientifold models following the ideas of [8].¹¹

Consider a set of N_p Dp -branes at a Γ orbifold singularity, where Γ is some finite group. The action of Γ on the Chan-Paton indices of the open strings ending on the Dp -branes is described by a (projective) representation γ that associates a $(N_p \times N_p)$ -matrix γ_g to each element g of Γ .

$$\gamma : \Gamma \longrightarrow GL(N_p, \mathbb{C}) \quad (\text{A.1})$$

$$g \longmapsto \gamma_g$$

In general, γ can be decomposed in a direct sum of irreducible representations (irreps) R_l :

$$\gamma = \bigoplus_l n_l R_l, \quad (\text{A.2})$$

where the notation $n_l R_l$ is short for $R_l \otimes \mathbb{1}_{n_l}$, i.e. n_l is the number of copies of the irrep R_l in γ . The action of Γ on the internal \mathbb{C}^3 is described by a representation R_{C^3} :

$$\gamma : R_{C^3} \longrightarrow SU(3) \quad (\text{A.3})$$

$$g \longmapsto R_{C^3}(g)$$

We write $R_{C^3} = R_{C^3}^{(1)} \oplus R_{C^3}^{(2)} \oplus R_{C^3}^{(3)}$ (this decomposition is possible whenever Γ is Abelian), where $R_{C^3}^{(i)}$ corresponds to the action of R_{C^3} on the i -th coordinate of \mathbb{C}^3 . Then the projection of the Chan-Paton matrices $\lambda^{(0)}$ (gauge fields) and $\lambda^{(i)}$ (matter fields) on Γ -invariant states reads:

$$\lambda^{(0)} = \gamma_g \lambda^{(0)} \gamma_g^{-1}, \quad \lambda^{(i)} = R_{C^3}^{(i)}(g) \gamma_g \lambda^{(i)} \gamma_g^{-1}. \quad (\text{A.4})$$

The solution of these equations leads to gauge group

$$G = \prod_l U(n_l) \quad (\text{A.5})$$

¹¹A detailed discussion of orbifold quivers can be found in [27] and the references therein. For a mathematical introduction to quiver theory, see e.g. [28].

and matter fields

$$\sum_{i=1}^3 \sum_{k,l} a_{kl}^{(i)} (\square_k, \bar{\square}_l), \quad (\text{A.6})$$

where the coefficients $a_{kl}^{(i)}$ only take the values 0 or 1 and are defined through

$$R_{C^3}^{(i)} \otimes R_k = \bigoplus_l a_{kl}^{(i)} R_l. \quad (\text{A.7})$$

Solving this equation for $a_{kl}^{(i)}$, one finds

$$a_{kl}^{(i)} = \frac{1}{|\Gamma|} \sum_{g \in \Gamma} R_{C^3}^{(i)}(g) \text{tr}(R_k(g)) \text{tr}(R_l(g^{-1})). \quad (\text{A.8})$$

Quiver diagrams are a nice graphical representation of the orbifold spectrum. To obtain the quiver corresponding to a Γ orbifold:

- determine all irreps R_l of Γ and associate a node \bullet to each irrep,
- calculate the coefficients $a_{kl}^{(i)}$ and draw an oriented link from the k -th to the l -th node $\bullet_k \rightarrow \bullet_l$ if $a_{kl}^{(i)} = 1$.

For $\Gamma = \mathbb{Z}_N$, there are N irreps, shown in eq. (2.8). It is easy to see that the coefficients determining the matter representations are $a_{kl}^{(i)} = \delta_{k, l+Nv_i \bmod N}$, where $v = (v_1, v_2, v_3)$ is the shift vector defined in (2.2). This leads to a quiver diagram similar to the one shown in figure 1.

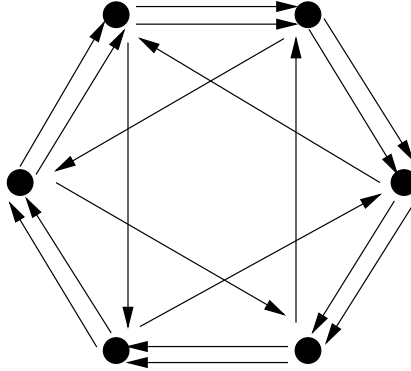


Figure 1: Quiver diagram of the \mathbf{Z}_6 orbifold with shift vector $v = \frac{1}{6}(1, 1, -2)$

The orientifold model is obtained by projecting the corresponding orbifold on Ω -invariant states. As a consequence, the representation γ of (A.1) must be real or pseudoreal. Again,

γ can be decomposed in irreducible blocks, as in (A.2). But now we have to distinguish between real, pseudoreal and complex irreps, eq. (2.4). Accordingly, we divide the set of indices l that label the irreps of Γ in three subsets $\{l\} = \mathcal{R} \cup \mathcal{P} \cup \mathcal{C}$. Due to the Ω -projection, the complex irreps contained in γ only appear in pairs of conjugate representations, i.e. if the l -th irrep is complex, then there exists an \bar{l} , such that $R_{\bar{l}} = \bar{R}_l$. It is convenient to divide the complex irreps into two subsets $\mathcal{C} = \mathcal{C}_1 \cup \mathcal{C}_2$, $\mathcal{C}_1 \cap \mathcal{C}_2 = \emptyset$, such that for each $l \in \mathcal{C}_1$ the conjugate index \bar{l} belongs to \mathcal{C}_2 and vice versa. In terms of quiver diagrams, this means that we have to identify nodes that correspond to conjugate representations. This leads to the following rules for orientifold quivers:

First draw a modified orbifold quiver diagram:

- determine all irreps of Γ and associate a node to each irrep depending on whether it is real \circ , pseudoreal \odot or complex \bullet ,
- compute the coefficients $a_{kl}^{(i)}$ using (A.8) and for each $a_{kl}^{(i)} = 1$ draw an oriented link from the k -th to the l -th node, e.g. $\bullet_k \rightarrow \bullet_l$ if $k, l \in \mathcal{C}$, $\circ_k \rightarrow \bullet_l$ if $k \in \mathcal{R}$, $l \in \mathcal{C}$, $l \in \mathcal{R}$, etc.

Then perform the Ω -projection on this quiver diagram (in the following, we only distinguish between \circ and \odot if they are non-equivalent):

- cancel the l -th node if $l \in \mathcal{C}_2$,
- cancel the link $\bullet_k \rightarrow \bullet_l$ if
 - $k, l \in \mathcal{C}_2$ or
 - $k \in \mathcal{C}_1$, $l \in \mathcal{C}_2$ and $k > \bar{l}$ or
 - $k \in \mathcal{C}_2$, $l \in \mathcal{C}_1$ and $\bar{k} > l$
- replace the link $\bullet_k \rightarrow \bullet_l$ by
 - $\bullet_k \rightarrow \bullet_l$ if $k \in \mathcal{C}_1$, $l \in \mathcal{C}_2$, $k < \bar{l}$
 - $\bullet_{\bar{k}} \rightarrow \bullet_l$ if $k \in \mathcal{C}_2$, $l \in \mathcal{C}_1$, $\bar{k} < l$
 - \bullet_k if $k \in \mathcal{C}_1$, $l \in \mathcal{C}_2$, $k = \bar{l}$
 - \bullet_l if $k \in \mathcal{C}_2$, $l \in \mathcal{C}_1$, $\bar{k} = l$
- cancel the link $\circ_k \rightarrow \bullet_l$ (or $\bullet_k \rightarrow \odot_l$) if $l \in \mathcal{C}_2$
- cancel the link $\circ_k \rightarrow \odot_l$ if $k > l$

- cancel all arrows (not the links) pointing towards a real node \bullet , i.e. replace $\bullet \xrightarrow{k} \circ_l$ by $\bullet \xrightarrow{k} \circ_l$, etc.

In Figure 2 we illustrate these rules by showing how the quiver diagram of the \mathbb{Z}_4 orientifold with vector structure is obtained.

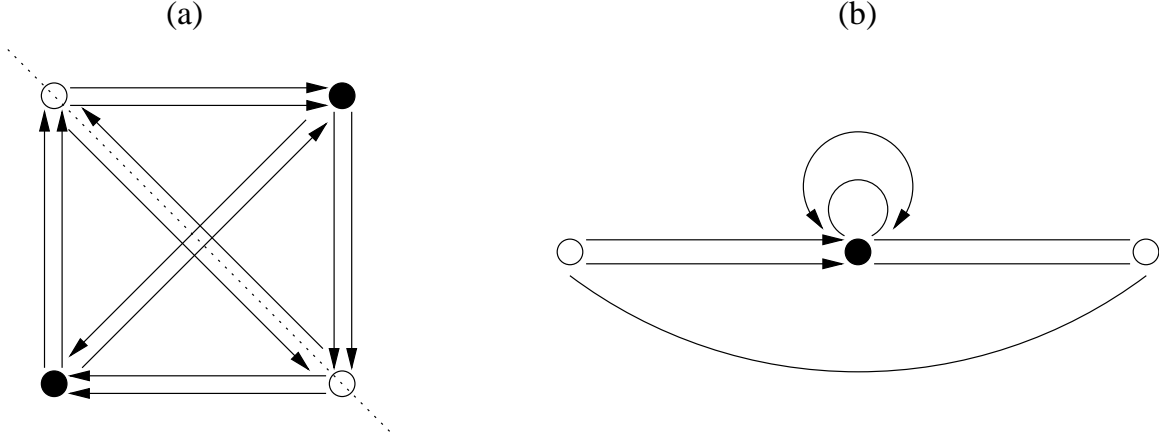


Figure 2: Quiver diagram (a) of the \mathbf{Z}_4 orbifold with shift vector $v = \frac{1}{4}(1, 1, -2)$ and (b) of the corresponding orientifold with vector structure. The dotted line indicates the axis along which the Ω -projection is performed.

The spectrum can easily be read off from the orientifold quiver. Each node corresponds to a gauge group factor:

$$\bullet_i : U(n_i), \quad \circ_i : \begin{cases} SO(n_i) & \text{if } c_p = 1 \\ USp(n_i) & \text{if } c_p = -1 \end{cases}, \quad \odot_i : \begin{cases} USp(n_i) & \text{if } c_p = 1 \\ SO(n_i) & \text{if } c_p = -1 \end{cases},$$

where $c_p = +1$ (-1) if $\gamma_{\Omega,p}$ is (anti)symmetric. Each link corresponds to a matter field transforming as a bifundamental (if it connects two different nodes) or as a second rank tensor (if it starts and ends on the same node). The representation is by definition fundamental at the tail of the link and antifundamental at the head of the link.

$$\begin{aligned} \bullet_k \xrightarrow{\quad} \bullet_l &: (\square_k, \bar{\square}_l), & \bullet_k \xrightarrow{\quad} \circ_l &: (\square_k, \square_l), & \circ_k \xrightarrow{\quad} \circ_l &: (\square_k, \square_l), & \dots \\ \bullet_k \xrightarrow{\quad} \bullet_k &: adj_k, & \bullet_k \xrightarrow{\quad} \bullet_k &: \begin{cases} \square_k & \text{if } c_p = 1 \\ \bar{\square}_k & \text{if } c_p = -1 \end{cases}, & \bullet_k \xrightarrow{\quad} \bullet_k &: \begin{cases} \bar{\square}_k & \text{if } c_p = 1 \\ \square_k & \text{if } c_p = -1 \end{cases} \end{aligned}$$

It turns out that it is more difficult to find the tensor representations of orthogonal or symplectic groups. We need to define

$$c_k^{(i)} = \frac{1}{|\Gamma|} \sum_{g \in \Gamma} R_{C^3}^{(i)}(g) \text{tr}(R_k(g^2)). \quad (\text{A.9})$$

Then, one has

$$\bigcirc_k : \begin{cases} \square_k & \text{if } c_p c_k^{(i)} = 1 \\ \square_k & \text{if } c_p c_k^{(i)} = -1 \end{cases}$$

Note that the index (A.9) is a direct generalization of the well-known Frobenius-Schur index

$$c_k^{\text{FS}} = \frac{1}{|\Gamma|} \sum_{g \in \Gamma} \text{tr}(R_k(g^2)) = \begin{cases} 1 & \text{if } R_k \text{ real} \\ -1 & \text{if } R_k \text{ pseudoreal} \\ 0 & \text{if } R_k \text{ complex} \end{cases} \quad (\text{A.10})$$

Applying these rules to the quiver diagram of the \mathbb{Z}_4 orientifold, figure 2, one immediately finds the spectrum of the 99 sector and the 55 sector displayed in table 3.

The above rules to determine the orientifold spectrum are easily generalized to also include the 95 sector. One has to calculate the coefficients a_{kl}^{95} defined by

$$a_{kl}^{95} = \frac{1}{|\Gamma|} \sum_{g \in \Gamma} \left(R_{C^3}^{(3)}(g) \right)^{1/2} \text{tr}(R_k(g)) \text{tr}(R_l(g^{-1})). \quad (\text{A.11})$$

By the arguments given at the beginning of this appendix, we find that the matter fields transforming in representations

$$\sum_{k,l} a_{kl}^{95} (\square_k, \bar{\square}_l), \quad (\text{A.12})$$

solve the projection equation (2.11). The first entry in the bifundamental in (A.12) corresponds to the transformation under the 99 gauge group and the second entry to the transformation under the 55 gauge group. This is already the orientifold spectrum because Ω only relates the 95 sector to the 59 sector but it imposes no new condition. Of course, one has to replace $(\square_k, \bar{\square}_l)$ by $(\bar{\square}_k, \bar{\square}_l)$ if $k \in \mathcal{C}_2$ and similarly if $l \in \mathcal{C}_2$.

B Tadpoles of \mathbb{Z}_N orientifolds

In this appendix, we use the strategy of [9, 29] to derive the tadpole cancellation conditions for \mathbb{Z}_N orientifolds. The idea is that the low-energy limit ($t \rightarrow 0$) of the one-loop amplitudes contributing to the tadpoles can be expressed as a sum over products of sines, cosines and traces of γ matrices. This method can be directly applied to the non-compact orientifolds. Two modifications arise in the case of compact orientifold models: First, the zeros or singularities of some sine- or cosine-factor have to be replaced by the appropriate volume factor V_i of the i -th internal torus. Second, the compactification leads to new fixed points, where new contributions to the tadpoles may appear. We will not consider the volume dependence of the four non-compact coordinates because it can be trivially factorized out.

In the following, we label the elements of \mathbb{Z}_N by the integer $k = 0, \dots, N-1$, and define $s_i = \sin(\pi k v_i)$, $c_i = \cos(\pi k v_i)$ and $\tilde{s}_i = \sin(2\pi k v_i)$.

The cylinder contribution to the tadpole cancellation conditions is:

$$\mathcal{C} = \sum_{k=0}^{N-1} \mathcal{C}_{(k)} = \sum_{k=0}^{N-1} \frac{1}{8s_1 s_2 s_3} [\text{Tr } \gamma_{k,9} + 4\alpha s_1 s_2 \text{Tr } \gamma_{k,5}]^2, \quad (\text{B.1})$$

where α is a sign related to the weight of the 95_3 sector. For supersymmetric models, it has been shown in [12] that $\alpha = -1$ if one uses the alternative action of Ω^2 on the oscillator ground state of the 95_3 sector, $\Omega^2|_{95_3} = +1$, whereas $\alpha = +1$ for the standard GP action.¹² The formula (B.1) is valid if there are no fixed tori. If the i -th internal torus is fixed, i.e. if $kv_i = 0 \bmod \mathbb{Z}$, one must take into account the volume dependence:

- if $kv_3 = 0 \bmod \mathbb{Z}$:

$$\mathcal{C}_{(k)} = \frac{V_3}{8s_1 s_2} [\text{Tr } \gamma_{k,9} + 4\alpha s_1 s_2 \text{Tr } \gamma_{k,5}]^2. \quad (\text{B.2})$$

- if $kv_{i \neq 3} = 0 \bmod \mathbb{Z}$:

$$\mathcal{C}_{(k)} = \mp \frac{V_i}{8s_3^2} [\text{Tr } \gamma_{k,9}]^2 \mp \frac{2}{V_i} [\text{Tr } \gamma_{k,5}]^2 + \frac{\alpha}{s_3} \text{Tr } \gamma_{k,9} \text{Tr } \gamma_{k,5}, \quad (\text{B.3})$$

where the upper (lower) sign refers to $kv_i = \text{even}$ (odd).

- if $k = 0$:

$$\mathcal{C}_{(0)} = \frac{V_1 V_2 V_3}{8} [\text{Tr } \gamma_{0,9}]^2 + \frac{2V_3}{V_1 V_2} [\text{Tr } \gamma_{0,5}]^2 + \alpha V_3 \text{Tr } \gamma_{0,9} \text{Tr } \gamma_{0,5}. \quad (\text{B.4})$$

The Klein bottle contribution can be split as:

$$\mathcal{K} = \sum_{k=0}^{N-1} (\mathcal{K}_0(k) + \mathcal{K}_1(k)), \quad (\text{B.5})$$

where \mathcal{K}_0 is the contribution of the untwisted sector and \mathcal{K}_1 is the contribution of the order-two sector (see [9, 6]). One has:

$$\mathcal{K}_0(k) = 16 \prod_{i=1}^3 \frac{2c_i^2}{\tilde{s}_i}, \quad \mathcal{K}_1(k) = 16 \epsilon \frac{2c_3^2}{\tilde{s}_3} \quad (\text{B.6})$$

The sign ϵ is related to the choice of the Ω -projection. One has $\epsilon = +1$ for the GP projection and $\epsilon = -1$ for the DPBZ projection, eq. (2.1). The Klein bottle contribution to the tadpole conditions can be reordered:

$$\mathcal{K} = \sum_{k=0}^{N/2-1} \frac{1}{8\tilde{s}_1 \tilde{s}_2 \tilde{s}_3} [32(c_1 c_2 c_3 + \epsilon s_1 s_2 c_3)]^2. \quad (\text{B.7})$$

¹²Non-supersymmetric models with GP action of Ω^2 on the 95_3 sector have $\alpha = -1$ in the RR tadpoles.

These formulae are valid if $2kv_i \neq 0 \bmod \mathbb{Z}$. If this is not the case, some of the sine- or cosine-factors have to be replaced by the appropriate volume factors. We find:

- if $kv_3 = 0$:

$$\mathcal{K}_0(k) + \mathcal{K}_1(k) = \frac{V_3}{8\tilde{s}_1\tilde{s}_2} [32(c_1c_2 + \epsilon s_1s_2)]^2. \quad (\text{B.8})$$

- if $kv_{i \neq 3} = 0$:

$$\mathcal{K}_0(k) + \mathcal{K}_1(k) = -\frac{V_i}{8\tilde{s}_3^2} [32c_3^2]^2 - \frac{2}{V_i} [8\epsilon]^2 + \frac{1}{\tilde{s}_3} [32c_3^2] [8\epsilon]. \quad (\text{B.9})$$

- if $kv_3 = \pm \frac{1}{2}$:

$$\mathcal{K}_0(k) + \mathcal{K}_1(k) = \frac{2}{V_3} [8(1 + \epsilon)]^2. \quad (\text{B.10})$$

- if $kv_{i \neq 3} = \pm \frac{1}{2}$:

$$\mathcal{K}_0(k) + \mathcal{K}_1(k) = \frac{V_i}{8\tilde{s}_3^2} [32c_3^2]^2 + \frac{2}{V_i} 8^2 + \frac{1}{\tilde{s}_3} [32c_3^2] [8\epsilon]. \quad (\text{B.11})$$

- if $k = 0$:

$$\mathcal{K}_0(0) + \mathcal{K}_1(0) = \frac{V_1V_2V_3}{8} 32^2 + \frac{2V_3}{V_1V_2} [32\epsilon]^2 - V_3 [32\epsilon] \cdot 32. \quad (\text{B.12})$$

The Möbius strip contribution can be split into the contributions from the $D9$ -branes and from the $D5_3$ -branes:

$$\mathcal{M}_9(k) = -8 \frac{1}{8s_1s_2s_3} \text{Tr} \left(\gamma_{\Omega k,9}^{-1} \gamma_{\Omega k,9}^\top \right), \quad (\text{B.13})$$

$$\mathcal{M}_5(k) = -8 \frac{2c_1c_2}{s_3} \alpha \Omega^2|_{95_3} \text{Tr} \left(\gamma_{\Omega k,5}^{-1} \gamma_{\Omega k,5}^\top \right),$$

where $\Omega^2|_{95_3}$ is defined in (2.14). There is a sign ambiguity in the Möbius strip amplitude. The factor $\alpha \Omega^2|_{95_3}$ in \mathcal{M}_5 has been introduced to reproduce the known tadpoles of the supersymmetric models of GP and DPBZ [11, 12, 13] and the non-supersymmetric models of AAADS and AU [19, 17]. The above equations can be rewritten as

$$\mathcal{M}_9 = -2 \sum_{k=0}^{N/2-1} \frac{c_9}{8\tilde{s}_1\tilde{s}_2\tilde{s}_3} [32(c_1c_2c_3 - \mu_9s_1s_2c_3)] [\text{Tr} \gamma_{2k,9}], \quad (\text{B.14})$$

$$\mathcal{M}_5 = -2 \sum_{k=0}^{N/2-1} \frac{-\tilde{c}_5}{8\tilde{s}_1\tilde{s}_2\tilde{s}_3} [32(c_1c_2c_3 - \mu_5s_1s_2c_3)] [4\tilde{s}_1\tilde{s}_2 \text{Tr} \gamma_{2k,5}],$$

where we defined $\tilde{c}_5 = -c_5 \alpha \Omega^2|_{95_3}$. For supersymmetric models, we have $\tilde{c}_5 = c_5$. The sign μ_p depends on whether the model has vector structure or not: $(\gamma_{1,p})^N = \mu_p \mathbb{I}$. The sign c_p tells us if $\gamma_{\Omega,p}$ is symmetric or antisymmetric: $\gamma_{\Omega,p}^\top = c_p \gamma_{\Omega,p}$. Again, these formulae are valid only if $2kv_i \neq 0 \bmod \mathbb{Z}$. Else one has to include the appropriate volume factors. We find:

- if $kv_3 = 0$:

$$\begin{aligned}\mathcal{M}_9(k) &= \frac{V_3}{8\tilde{s}_1\tilde{s}_2} 2[\text{Tr } \gamma_{2k,9}] [-32 c_9 (c_1 c_2 - \mu_9 s_1 s_2)], \\ \mathcal{M}_5(k) &= \frac{V_3}{8\tilde{s}_1\tilde{s}_2} 2[4\tilde{s}_1\tilde{s}_2 \text{Tr } \gamma_{2k,5}] [32 \tilde{c}_5 (c_1 c_2 - \mu_5 s_1 s_2)].\end{aligned}\tag{B.15}$$

- if $kv_{i \neq 3} = 0$:

$$\begin{aligned}\mathcal{M}_9(k) &= -\frac{V_i}{8\tilde{s}_3^2} 2[\text{Tr } \gamma_{2k,9}] [-32 c_9 c_3^2] + \frac{1}{\tilde{s}_3} [\text{Tr } \gamma_{2k,9}] [8 c_9 \mu_9], \\ \mathcal{M}_5(k) &= -\frac{2}{V_i} 2[\text{Tr } \gamma_{2k,5}] [-8 \tilde{c}_5 \mu_5] + \frac{1}{\tilde{s}_3} [\text{Tr } \gamma_{2k,5}] [32 \tilde{c}_5 c_3^2].\end{aligned}\tag{B.16}$$

- if $kv_3 = \pm \frac{1}{2}$:

$$\begin{aligned}\mathcal{M}_9(k) &= \frac{1}{\tilde{s}_1} [\text{Tr } \gamma_{2k,9}] [\mp 8 c_9 (1 - \mu_9)], \\ \mathcal{M}_5(k) &= \frac{1}{\tilde{s}_1} [4\tilde{s}_1\tilde{s}_2 \text{Tr } \gamma_{2k,5}] [\pm 8 \tilde{c}_5 (1 - \mu_5)].\end{aligned}\tag{B.17}$$

- if $kv_{i \neq 3} = \pm \frac{1}{2}$:

$$\begin{aligned}\mathcal{M}_9(k) &= \frac{V_i}{8\tilde{s}_3^2} 2[\text{Tr } \gamma_{2k,9}] [\pm 32 c_9 \mu_9 c_3^2] + \frac{1}{\tilde{s}_3} [\text{Tr } \gamma_{2k,9}] [\mp 8 c_9], \\ \mathcal{M}_5(k) &= \frac{2}{V_i} 2[\text{Tr } \gamma_{2k,5}] [\pm 8 \tilde{c}_5] + \frac{1}{\tilde{s}_3} [\text{Tr } \gamma_{2k,5}] [\mp 32 \tilde{c}_5 \mu_5 c_3^2].\end{aligned}\tag{B.18}$$

- if $k = 0$:

$$\begin{aligned}\mathcal{M}_9(0) &= \frac{V_1 V_2 V_3}{8} 2[\text{Tr } \gamma_{0,9}] [-32 c_9], \\ \mathcal{M}_5(0) &= \frac{2V_3}{V_1 V_2} 2[\text{Tr } \gamma_{0,5}] [-32 \tilde{c}_5 \mu_5].\end{aligned}\tag{B.19}$$

Finally, we want to write the sum of all contributions in a factorized form:

$$\mathcal{C} + \mathcal{M} + \mathcal{K} = \sum_{\text{odd } k} [\dots]^2 + \sum_{k=0}^{N/2-1} [\dots]^2.\tag{B.20}$$

From the above equations, we see that this factorization is only possible if

$$\epsilon = -\mu_9 = -\mu_5.\tag{B.21}$$

This means that models with vector structure are only possible if one uses the alternative Ω -projection of DPBZ (corresponding to $\epsilon = -1$). Assuming these conditions, the tadpoles take the following form:

a) untwisted sector

$$\frac{V_1 V_2 V_3}{8} [\text{Tr } \gamma_{0,9} - 32 c_9]^2 + \frac{2V_3}{V_1 V_2} [\text{Tr } \gamma_{0,5} - 32 \tilde{c}_5 \mu_5]^2 + V_3 [\alpha \text{Tr } \gamma_{0,9} \text{Tr } \gamma_{0,5} - 32^2 \epsilon]. \quad (\text{B.22})$$

b) twisted sectors without fixed tori, i.e. $kv_i \neq 0 \bmod \mathbb{Z}$:

• odd k :

$$\frac{1}{8s_1 s_2 s_3} [\text{Tr } \gamma_{k,9} + 4\alpha s_1 s_2 \text{Tr } \gamma_{k,5}]^2. \quad (\text{B.23})$$

• even $k = 2k'$:

$$\frac{1}{8\tilde{s}_1 \tilde{s}_2 \tilde{s}_3} [\text{Tr } \gamma_{2k',9} + 4\alpha \tilde{s}_1 \tilde{s}_2 \text{Tr } \gamma_{2k',5} - 32c_9 (c_1 c_2 c_3 + \epsilon s_1 s_2 c_3)]^2, \quad (\text{B.24})$$

where s_i, c_i, \tilde{s}_i are evaluated with the argument k' .

c) twisted sectors with fixed tori, i.e. $kv_i = 0 \bmod \mathbb{Z}$:

• odd k :

- $i = 3$:

$$\frac{V_3}{8s_1 s_2} [\text{Tr } \gamma_{k,9} + 4\alpha s_1 s_2 \text{Tr } \gamma_{k,5}]^2. \quad (\text{B.25})$$

- $i \neq 3$: never happens

• even $k = 2k'$, with $k'v_i = 0$:

- $i = 3$:

$$\frac{V_3}{8\tilde{s}_1 \tilde{s}_2} [\text{Tr } \gamma_{2k',9} + 4\alpha \tilde{s}_1 \tilde{s}_2 \text{Tr } \gamma_{2k',5} - 32c_9 (c_1 c_2 + \epsilon s_1 s_2)]^2. \quad (\text{B.26})$$

- $i \neq 3$:

$$\begin{aligned} & -\frac{V_i}{8\tilde{s}_3^2} [\text{Tr } \gamma_{2k',9} + 32c_9 c_3^2]^2 - \frac{2}{V_i} [\text{Tr } \gamma_{2k',5} + 8\tilde{c}_5 \mu_5]^2 \\ & + \frac{\alpha}{\tilde{s}_3} [\text{Tr } \gamma_{2k',9} + 32c_9 c_3^2] [\text{Tr } \gamma_{2k',5} + 8\tilde{c}_5 \mu_5]. \end{aligned} \quad (\text{B.27})$$

• even $k = 2k'$, with $k'v_i = \pm \frac{1}{2}$:

- $i = 3$:

$$\begin{aligned} & \frac{V_3}{8\tilde{s}_1\tilde{s}_2} [\text{Tr } \gamma_{2k',9} + 4\alpha \tilde{s}_1\tilde{s}_2 \text{Tr } \gamma_{2k',5}]^2 + \frac{2}{V_3} [8(1+\epsilon)]^2 \\ & \mp \frac{c_9}{\tilde{s}_1} [\text{Tr } \gamma_{2k',9} + 4\alpha \tilde{s}_1\tilde{s}_2 \text{Tr } \gamma_{2k',5}] [8(1+\epsilon)]. \end{aligned} \quad (\text{B.28})$$

- $i \neq 3$:

$$\begin{aligned} & \frac{V_i}{8\tilde{s}_3^2} [\text{Tr } \gamma_{2k',9} \pm 32c_9\mu_9c_3^2]^2 + \frac{2}{V_i} [\alpha \text{Tr } \gamma_{2k',5} \pm 8\tilde{c}_5]^2 \\ & + \frac{\alpha}{\tilde{s}_3} [\text{Tr } \gamma_{2k',9} \pm 32c_9\mu_9c_3^2] [\text{Tr } \gamma_{2k',5} \pm 8\tilde{c}_5]. \end{aligned} \quad (\text{B.29})$$

From the form of the untwisted tadpoles (B.22), we find that a cancellation is only possible if

$$c_9 = 1, \quad \epsilon = \alpha. \quad (\text{B.30})$$

The first condition could be evaded by introducing anti- $D9$ -branes. Indeed, when fixing the sign of the Möbius strip amplitude, eq. (B.13), we implicitly assumed that anti-branes may only appear in the 5-brane sector. However, the second condition in (B.30) is always valid. It means that D -branes and O -planes must have opposite charges.

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